26.1 Examples

1. A wind turbine has to be brought to a stop for maintenance. At the time the brakes are initially applied, the turbine is rotating at 2 radians per second. During the braking, the angular acceleration of the turbine is given by

$$\alpha = -0.015\sqrt{1+5t}\ \text{rad/s}^2.$$

How long does it take for the turbine to come to a complete stop?

Recall that rotating objects have angular acceleration $\alpha = \dfrac{d\omega}{dt}$.

Plan: ① $w = \int \alpha\, dt = \_\_\_\_\_\_ + C$

find C using $w = 2$ when $t = 0$

② solve for t when $w = 0$

$w = \int \alpha\, dt = \int -0.015\,(\underbrace{1+5t}_{\text{linear}})^{1/2}\, dt$ or use $u = 1+5t$

$= -0.015 \cdot \frac{1}{5} \cdot \frac{2}{3}(1+5t)^{3/2} + C$

$w = -0.002\,(1+5t)^{3/2} + C$

$w = 2$ when $t = 0$

$2 = -0.002\,(1+5(0))^{3/2} + C$

$2 = -0.002 + C$

$C = 2.002$

so $w = -0.002\,(1+5t)^{3/2} + 2.002$

Now we can solve for t when $w = 0$
complete stop

$0 = -0.002\,(1+5t)^{3/2} + 2.002$

$-2.002 = -0.002\,(1+5t)^{3/2}$

$[1001]^{2/3} = [(1+5t)^{3/2}]^{2/3}$

$1001^{2/3} = 1+5t$

$1001^{2/3} - 1 = 5t$

$t = \dfrac{1001^{2/3} - 1}{5} \approx \boxed{19.81\ \text{s}}$

2. As a rocket burns, it consumes fuel and consequently gets lighter in mass. If a Saturn V rocket initially starts out with 2×10^6 kg of fuel and the rate of change of the mass of fuel is given by

$$\frac{dm}{dt} = -t\sqrt{t^2+100} \text{ kg/s},$$

how long does it take to burn all the fuel?

i.e. Find the value of t for which the mass of fuel is zero.

Plan: ① find m using

$$m = \int \frac{dm}{dt}\, dt = \underline{\qquad\qquad} + C$$

use $m = 2\times10^6$ when $t=0$

② find t when $m=0$

$$m = \int -t\,(\underbrace{t^2+100}_{u})^{1/2}\, dt$$

$\frac{1}{2}du$

$u = t^2+100$

$du = 2t\, dt$

$\frac{1}{2}du = t\,dt$

$$= -\frac{1}{2}\int u^{1/2}\, du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$m = -\frac{1}{3}(t^2+100)^{3/2} + C$$

$m = 2\times10^6$ when $t=0$

$$2\times10^6 = -\frac{1}{3}(0^2+100)^{3/2} + C$$

$$2\times10^6 = -\frac{1}{3}(1000) + C$$

$$C = 2\times10^6 + \frac{1000}{3}$$

so $m = -\frac{1}{3}(t^2+100)^{3/2} + 2\times10^6 + \frac{1000}{3}$

now solve for t when $m=0$

$$0 = -\frac{1}{3}(t^2+100)^{3/2} + 2\times10^6 + \frac{1000}{3}$$

$$\frac{1}{3}(t^2+100)^{3/2} = 2\times10^6 + \frac{1000}{3}$$

$$\left[(t^2+100)^{3/2}\right]^{2/3} = \left[6\times10^6 + 1000\right]^{2/3}$$

$$t^2+100 = 6001000^{2/3}$$

$$t^2 = 6001000^{2/3} - 100 \quad \text{and } t>0$$

$$t = \sqrt{6001000^{2/3} - 100} \approx \boxed{181.45\text{s}}$$