## 26.6 Other Applications of the Definite Integral

1. **Work** done by a variable force

   If $F(x)$ is the force at position $x$, then the work done by this force from $x = a$ to $x = b$ is

   $$w = \int_a^b F(x)\, dx$$

   **Example:** The force required to stretch a spring $x$ units from its unstretched length (aka its equilibrium position) is given by Hooke's Law:

   $$F(x) = kx,$$

   where $k$ is a constant that depends on the specific spring.

   It takes a force of 5 N to stretch a spring 1 cm from its equilibrium. Find the work done in stretching this spring from $x = 0$ to $x = 2$ cm.

2. **Average Value** of a Function

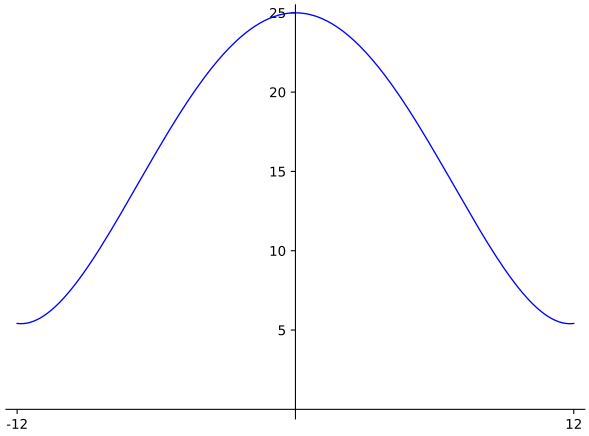
The average value of a function $f(x)$ on the interval $[a, b]$ is

$$y_{av} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

**Example:** The temperature $T$ (in °C) recorded in a city during a given day approximately followed the curve of

$$T = 0.001t^4 - 0.280t^2 + 25.0,$$

where $t$ is the number of hours from noon ($-12 \leq t \leq 12$). What was the average temperature during the day?

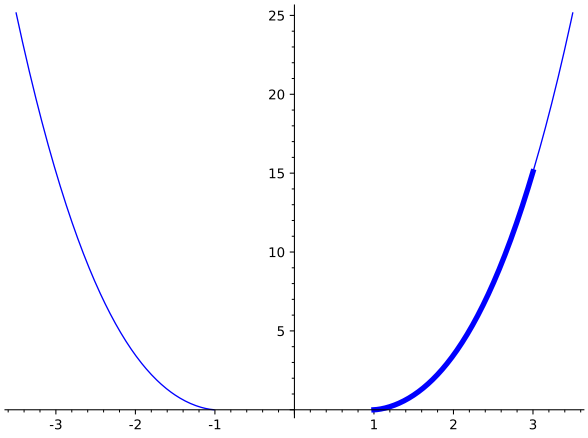


3. **Arc Length** on a Graph

The length of the curve on the graph of $y$ from $x = a$ to $x = b$ is

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

**Example:** Find the arc length of $y = \frac{2}{3}(x^2 - 1)^{3/2}$ on the interval $[1, 3]$.



4. **Surface Area** of a Solid of Revolution

The lateral surface area of a solid of revolution generated by rotating the region under the graph of $y$ between $x = a$ and $x = b$ around the $x$-axis is

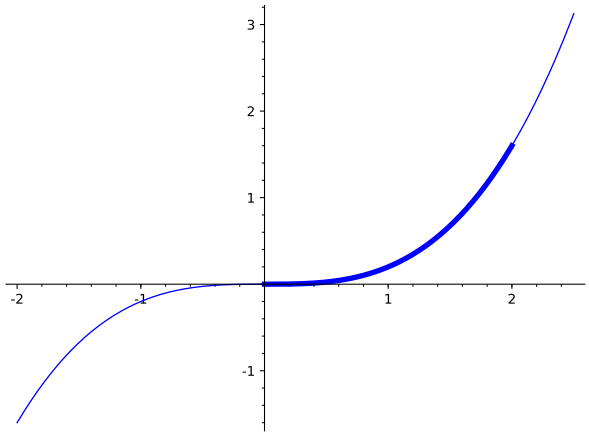
$$SA = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

Note: this does not include the vertical circles at $x = a$ and $x = b$. To include those, add

$$\pi(f(a))^2 + \pi(f(b))^2$$

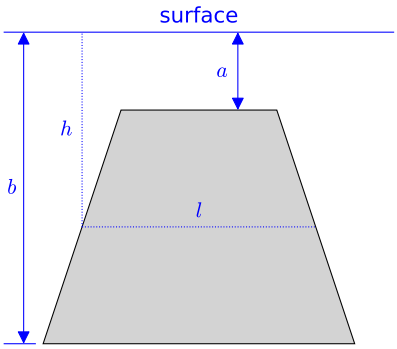
to the lateral surface area.

**Example:** The grinding surface of a grinding machine can be described as the surface generated by rotating the graph of $y = 0.2x^3$ from $x = 0$ to $x = 2$ around the $x$-axis. Find the grinding surface area.



5. **Force** due to Liquid Pressure

The force due to liquid pressure on a plate submerged vertically in a liquid of density $\gamma$ is given by:



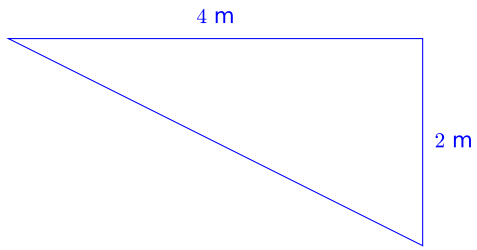

$$F = \gamma \int_a^b hl \, dh,$$

where the length $l$ is a function of the depth $h$.

Note that the density of water is $\gamma = 9800$ N/m$^3$.

**Example 1:** A rectangular floodgate of a dam is 3 m wide by 2 m high. Find the force on the vertical floodgate if its upper edge is 1 m below the surface of the water.

**Example 2:** A vertical end of a tank full of water is in the shape of a right triangle:




What is the force on the end of the tank?