The Derivatives of Sine and Cosine

Complete the following tables of values. Round off to at least 5 decimal places.

RADIANS

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$y = \dfrac{\sin h}{h}$						

degrees

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$y = \dfrac{\sin h}{h}$						

RADIANS

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$y = \dfrac{\cos h - 1}{h}$						

degrees

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$y = \dfrac{\cos h - 1}{h}$						

So in RADIANS, $\lim\limits_{h \to 0} \dfrac{\sin h}{h} =$ and $\lim\limits_{h \to 0} \dfrac{\cos h - 1}{h} =$

Also recall that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

For $f(x) = \sin x$ in RADIANS,

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} \\
&= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin(x)}{h} \\
&= \lim_{h \to 0} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h} \\
&= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} \\
&= \lim_{h \to 0} \left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}\right] \\
&= \lim_{h \to 0} \left[\sin x \cdot \frac{\cos h - 1}{h} + \cos x \cdot \frac{\sin h}{h}\right] \\
&= \sin x \cdot 0 + \cos x \cdot 1 \\
&= \cos x.
\end{aligned}$$

To find the derivative of the cosine function, recall that

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

and that

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

Furthermore, the *chain rule* for the derivative of a composition of functions is

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x).$$

As a result, in RADIANS

$$\begin{aligned}
\frac{d}{dx}(\cos x) &= \frac{d}{dx}\left(\sin\left(x + \frac{\pi}{2}\right)\right) \\
&= \cos\left(x + \frac{\pi}{2}\right) \cdot 1 \\
&= \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} \\
&= \cos x \cdot 0 - \sin x \cdot 1 \\
&= -\sin x.
\end{aligned}$$