27.1 and 27.2 Trig Derivatives

Differentiate the following:

1. $y = \underbrace{\sin x}_{u}\underbrace{\cos x}_{v}$ product rule

$y' = uv' + vu' = \sin x(-\sin x) + \cos x(\cos x)$

$= -\sin^2 x + \cos^2 x$

2. $y = 3\cos(4x)$

$y' = 3 \cdot (-\sin 4x) \cdot 4$

$= -12\sin 4x$

3. $y = \sin^4 x = (\sin x)^4$

$y' = 4(\sin x)^3 \cdot \cos x$

$= 4\sin^3 x \cos x$

4. $y = \sin(x^4)$

$y' = \cos(x^4) \cdot 4x^3$

$= 4x^3 \cos x^4$

5. $y = \sin^4 x^4 = [\sin(x^4)]^4$

$y' = 4[\sin(x^4)]^3 \cdot \cos(x^4) \cdot 4x^3$

$= 16x^3 \sin^3(x^4)\cos(x^4)$

6. $y = 3\sin^3(2x^4 + 1) = 3[\sin(2x^4+1)]^3$

$y' = 3 \cdot 3[\sin(2x^4+1)]^2 \cdot \cos(2x^4+1) \cdot 8x^3$

$= 72x^3 \sin^2(2x^4+1)\cos(2x^4+1)$

7. $y = (x - \cos^2 x)^4$

$(\cos x)^2$

$y' = 4(x - \cos^2 x)^3 \cdot (1 - 2\cos x \cdot (-\sin x))$

$= 4(x - \cos^2 x)^3 (1 + 2\cos x \sin x)$

8. $y = \cos^3 4x \sin^2 2x$ $= (\cos(4x))^3 \cdot (\sin(2x))^2$ product rule

$y' = uv' + vu' = \cos^3 4x \cdot 2(\sin(2x)) \cdot \cos(2x) \cdot 2 + \sin^2 2x \cdot 3(\cos(4x))^2(-\sin(4x)) \cdot 4$

$= 4\cos^3 4x \sin 2x \cos 2x - 12 \sin^2 2x \cos^2 4x \sin 4x$

$= 4\cos^3 4x \sin 2x (\cos 4x \cos 2x - 3 \sin 2x \sin 4x)$

9. $y = 3\tan(3x + 2)$

$y' = 3\sec^2(3x+2) \cdot 3$

$= 9\sec^2(3x+2)$

10. $y = 3\cot(6x)$

$y' = 3(-\csc^2(6x)) \cdot 6$

$= -18\csc^2 6x$

11. $y = \csc\sqrt{1-x}$ $= \csc(1-x)^{1/2}$ ② ③ ①

$y' = -\csc(1-x)^{1/2}\cot(1-x)^{1/2} \cdot \frac{1}{2}(1-x)^{-1/2} \cdot (-1)$

$= \dfrac{\csc(1-x)^{1/2}\cot(1-x)^{1/2}}{2(1-x)^{1/2}}$

12. $y = \frac{1}{2}\sin(2x)\sec x$ product rule

$y' = uv' + vu' = \frac{1}{2}\sin 2x \cdot \sec x \tan x + \sec x \cdot \frac{1}{2}\cos(2x) \cdot 2$

$= \frac{1}{2}\sin 2x \sec x \tan x + \sec x \cos 2x$