27.1 to 27.3 Derivatives of Trig and Inverse Trig Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Differentiate the following:

1. $y = 4\sec(1-x^3)$

$$y' = 4 \cdot \sec(1-x^3)\tan(1-x^3) \cdot (-3x^2)$$

$$= -12x^2 \sec(1-x^3)\tan(1-x^3)$$

2. $y = 0.4\cos^{-1}(2\pi x + 1)$

$$y' = 0.4 \cdot \frac{-1}{\sqrt{1-(2\pi x+1)^2}} \cdot 2\pi = \frac{-0.8\pi}{\sqrt{1-(2\pi x+1)^2}}$$

3. $y = \sin(\tan^{-1} x)$

$$y' = \cos(\tan^{-1} x) \cdot \frac{1}{1+x^2} = \frac{\cos(\tan^{-1} x)}{1+x^2}$$

4. $y = 3\cos^2(\tan 3x)$ $= 3\left[\cos(\tan(3x))\right]^2$

$$y' = 3\cdot 2\left[\cos(\tan 3x)\right]\cdot\left[-\sin(\tan 3x)\right]\cdot \sec^2 3x\cdot 3$$
$$= -18\cos(\tan 3x)\sin(\tan 3x)\sec^2 3x$$

5. $y = \dfrac{\sin^{-1}x}{4x}$ $\begin{matrix} u \\ v\end{matrix}$

$$y' = \frac{vu' - uv'}{v^2} = \frac{4x\cdot\frac{1}{\sqrt{1-x^2}} - \sin^{-1}x\cdot 4}{(4x)^2}$$

$$= \frac{\cancel{4}x - \cancel{4}\sin^{-1}x\cdot\sqrt{1-x^2}}{\sqrt{1-x^2}}\cdot\frac{1}{4\,\cancel{16}x^2} = \frac{x - \sqrt{1-x^2}\,\sin^{-1}x}{4x^2\sqrt{1-x^2}}$$

6. $y = x\cos^{-1}(x) - \sqrt{1-x^2}$ $= \underbrace{x}_{u}\underbrace{\cos^{-1}x}_{v} - (1-x^2)^{1/2}$

$$y' = x\cdot\frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x\cdot 1 - \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)$$

$$= \cancel{\frac{-x}{\sqrt{1-x^2}}} + \cos^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} = \cos^{-1}x$$

7. $y = \underbrace{x}_{u}\underbrace{(\tan^{-1}x)^2}_{v} - 2x$

$$y' = x\cdot 2(\tan^{-1}x)\cdot\frac{1}{1+x^2} + (\tan^{-1}x)^2\cdot 1 - 2$$

$$= \frac{2x\tan^{-1}x}{1+x^2} + (\tan^{-1}x)^2 - 2$$