27.5 and 27.6 Derivatives of Logarithmic and Exponential Functions

Differentiate the following:

1. $y = \log_3(7x)$

$$y' = \frac{1}{(\cancel{7}x)\ln 3} \cdot \cancel{7} = \frac{1}{x \ln 3}$$

2. $y = \ln(2x^3 - 4)$

$$y' = \frac{1}{2x^3-4} \cdot 6x^2 = \frac{6x^2}{2(x^3-2)} = \frac{3x^2}{x^3-2}$$

3. $y = \ln(3x^2 + 5x - 1)$

$$y' = \frac{1}{3x^2+5x-1} \cdot (6x+5) = \frac{6x+5}{3x^2+5x-1}$$

4. $y = \ln(\sec^2 x) = 2\ln(\sec x)$

$$y' = 2 \cdot \frac{1}{\cancel{\sec x}} \cdot \cancel{\sec x}\tan x = 2\tan x$$

OR $y' = 2 \cdot \frac{1}{\sec^2 x} \cdot 2(\sec x) \cdot \sec x \tan x = 2\tan x$

5. $y = \ln(5x^2 - 3)^4$

$= 4\ln(5x^2-3)$

$$y' = 4 \cdot \frac{1}{5x^2-3} \cdot 10x = \frac{40x}{5x^2-3}$$

6. $y = 3x^6 \ln\sqrt{x} = 3x^6 \cdot \ln x^{1/2} = 3x^6 \cdot \frac{1}{2}\ln x = \underbrace{\frac{3}{2}x^6}_{u}\underbrace{\ln x}_{v}$

$$y' = \frac{3}{2}x^6 \cdot \frac{1}{x} + \ln x \cdot 9x^5$$

$$= \frac{3}{2}x^5 + 9x^5 \ln x = 3x^5\left(\frac{1}{2} + 3\ln x\right)$$

$y = \underbrace{3x^6}_{u} \cdot \underbrace{\ln x^{1/2}}_{v}$

$y' = 3x^6 \cdot \frac{1}{x^{1/2}} \cdot \frac{1}{2}x^{-1/2} + \ln x^{1/2} \cdot 18x^5$

$= \dots = 3x^5\left(\frac{1}{2} + 6\ln x^{1/2}\right)$

7. $y = \cos^{-1}(\ln x^2)$ $= \cos^{-1}(2\ln x)$

$$y' = \frac{-1}{\sqrt{1-(2\ln x)^2}} \cdot 2 \cdot \frac{1}{x} = \frac{-2}{x\sqrt{1-(2\ln x)^2}} = \frac{-2}{x\sqrt{1-(\ln x^2)^2}}$$

8. $y = 10^{x^2}$

$$y' = 10^{x^2} \ln 10 \cdot 2x = (2\ln 10)\, x\, 10^{x^2}$$

9. $y = \frac{7}{e^x}$ $= 7e^{-x}$

$$y' = 7e^{-x}(-1) = \frac{-7}{e^x}$$

10. $y = 5\sqrt{x}e^{\pi x}$ $= \underbrace{5x^{1/2}}_{u}\underbrace{e^{\pi x}}_{v}$

$$y' = 5x^{1/2} \cdot e^{\pi x} \cdot \pi + e^{\pi x} \cdot \frac{5}{2} x^{-1/2} = 5e^{\pi x}\left(\pi x^{1/2} + \frac{1}{2x^{1/2}}\right)$$

$$= 5e^{\pi x}\left(\frac{2\pi x + 1}{2x^{1/2}}\right)$$

11. $y = \underbrace{4\ln(e^x+2)}_{u}\underbrace{\sin(\tfrac{1}{2}x)}_{v}$

$$y' = 4\ln(e^x+2) \cdot \cos(\tfrac{1}{2}x) \cdot \tfrac{1}{2} + \sin(\tfrac{1}{2}x) \cdot 4 \cdot \frac{1}{e^x+2} \cdot e^x$$

$$= 2\ln(e^x+2)\cos\tfrac{1}{2}x + \frac{4e^x \sin\frac{1}{2}x}{e^x+2}$$

12. $y = \underbrace{0.3e^{5x}}_{u}\underbrace{\ln(\csc x)}_{v}$

$$y' = 0.3e^{5x}\frac{1}{\cancel{\csc x}} \cdot (-\cancel{\csc x}\cot x) + \ln(\csc x) \cdot 0.3e^{5x} \cdot 5 = -0.3e^{5x}\cot x + 1.5e^{5x}\ln(\csc x)$$

$$= -0.3e^{5x}(\cot x - 5\ln(\csc x))$$

13. Use Newton's method to find the solution of $e^x = \sin x$ between -4 and -3.

$e^x - \sin x = 0$

so $f(x) = e^x - \sin x$

$f'(x) = e^x - \cos x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{(e^{x_n} - \sin x_n)}{(e^{x_n} - \cos x_n)}$$

RADS

$x_1 = -3.5$

$x_2 = -3.16836$

$x_3 = -3.18306$

$x_4 = -3.18306$