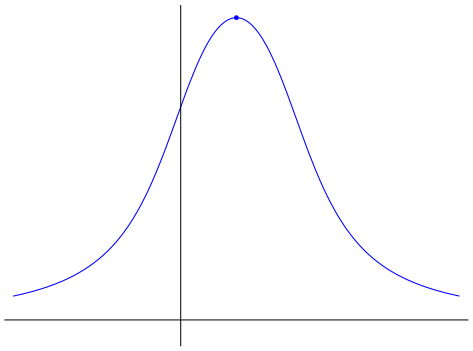
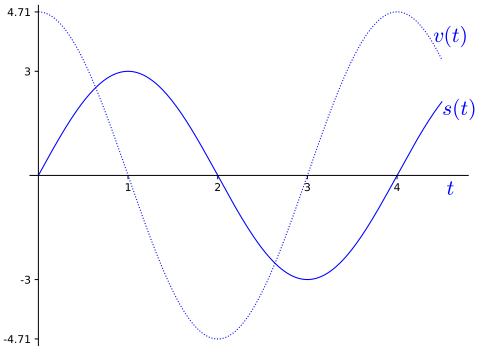
27.8 Applications

1. Find the maximum value of the function $f(x) = \tan^{-1} x + \tan^{-1}(2 - x)$.
   Note that the domain of this function is $\mathbb{R}$.



2. The displacement (in cm) of an object moving up and down in a straight line is given by $s(t) = 3\sin\left(\frac{\pi}{2}t\right)$, where t is in seconds.

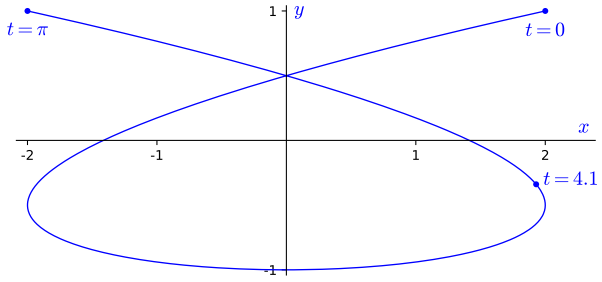


   (a) Complete the following table using the graphs above:

t (in s)	$s(t)$ (in cm)	$v(t)$ (in cm/s)	direction of motion (up/down/switching)
1			
2			
3			
4			

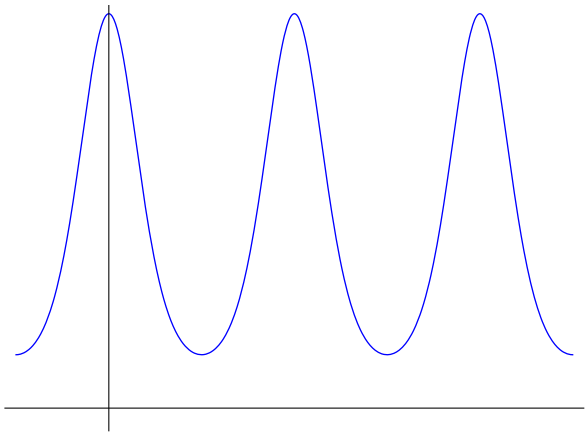
   (b) Find the velocity and direction of motion at $t = 2.5$ s.

3. A machine is programmed to move an etching tool such that the position (in cm) of the tool is given by $x = 2\cos 3t$ and $y = \cos 2t$, where t is the time (in s). Find the velocity of the tool for $t = 4.1$ s.

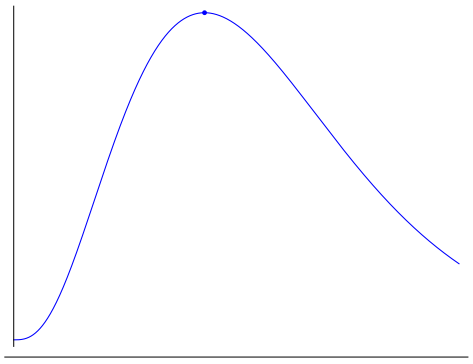


4. A car passes directly under a hovering police helicopter 150 m above a straight and level highway. After the car has traveled another 20 m, the angle of depression of the car from the helicopter is decreasing at the rate of 0.215 rad/s. What is the speed of the car?

5. Find the minimum value of the function $f(x) = e^{\cos x}$ on the interval $0 < x < 2\pi$. Give your answer in both exact and decimal form.

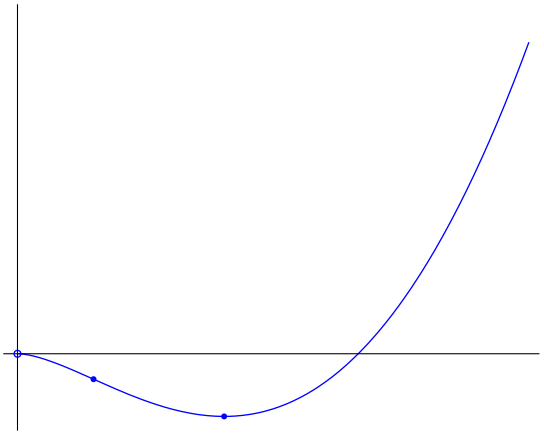


6. In a study of traffic control, the number n of vehicles on a certain section of a highway from 2 PM to 8 PM was found to be $n = 200(1 + t^3e^{-t})$, where t is the number of hours after 2 PM. At what time is the number of vehicles the greatest?



7. (a) Find the coordinates of the point of inflection and the minimum on the graph of

$$f(x) = x^2 \ln x.$$



(b) Find the equation of the tangent line to this graph at the x-intercept $(1, 0)$.

(c) Use the equation of the tangent line from part (b) to estimate the value of $f(0.9)$.

Answers:

1. $\frac{\pi}{2} \simeq 1.57$ rad
2. (a)

t	$s(t)$	$v(t)$	direction of motion
1	3	0	switching
2	0	-4.71	down
3	-3	0	switching
4	0	4.71	up

   (b) -3.33 cm/s, down
3. 2.46 cm/s @ 310.01°
4. 32.82 m/s
5. $\frac{1}{e} \simeq 0.37$
6. 5 PM
7. (a) poi $(\frac{1}{e^{3/2}}, -\frac{3}{2e^3}) \simeq (0.22, -0.07)$

      minimum $(\frac{1}{\sqrt{e}}, -\frac{1}{2e}) \simeq (0.61, -0.18)$

   (b) $y = x - 1$

   (c) -0.1