Chapter 27 Review

Find the derivatives of the following:

1. $y = 3x^4 - 2x^3 + \pi x + e$

$$y' = 12x^3 - 6x^2 + \pi$$

2. $f(x) = \underbrace{\sin(2x)}_{u}\,\underbrace{\ln(x-1)}_{v}$

$$f'(x) = uv' + vu' = \sin(2x) \cdot \frac{1}{x-1} \cdot 1 + \ln(x-1) \cdot \cos(2x) \cdot 2$$

$$= \frac{\sin 2x}{x-1} + 2\cos 2x \ln(x-1)$$

3. $f(x) = \cos^4(x^3 - 2)$

$$= \left(\cos(x^3-2)\right)^4$$

$$f'(x) = 4\left(\cos(x^3-2)\right)^3 \cdot \left(-\sin(x^3-2)\right) \cdot 3x^2$$

$$= -12x^2 \cos^3(x^3-2)\sin(x^3-2)$$

4. $y = \dfrac{\sqrt{2x^2-3}}{4^x} = \dfrac{(2x^2-3)^{1/2}}{4^x}$ $\begin{matrix} u \\ v \end{matrix}$

$$y' = \frac{vu' - uv'}{v^2} = \frac{\cancel{4^x} \cdot \frac{1}{\cancel{2}}(2x^2-3)^{-1/2} \cdot \cancel{4}^{\,2}x - (2x^2-3)^{1/2} \cdot \cancel{4^x} \ln 4}{(4^x)^{\cancel{2}}}$$

$$= \frac{2x - (2x^2-3)^{1/2} \cdot (2x^2-3)^{1/2} \ln 4}{(2x^2-3)^{1/2}} \cdot \frac{1}{4^x} = \frac{2x - (2x^2-3)\ln 4}{(2x^2-3)^{1/2}\, 4^x}$$

5. $f(x) = \underbrace{e^{4x}}_{u}\,\underbrace{\tan^{-1}(x^3+x)}_{v}$

$$f'(x) = uv' + vu' = e^{4x} \cdot \frac{1}{1+(x^3+x)^2} \cdot (3x^2+1) + \tan^{-1}(x^3+x) \cdot e^{4x} \cdot 4$$

$$= e^{4x}\left(\frac{3x^2+1}{1+(x^3+x)^2} + 4\tan^{-1}(x^3+x)\right)$$

6. $y = \underbrace{2x^4}_{u}\underbrace{\sec(e^x - 1)}_{v} + \sin^{-1}(5x)$

$$y' = 2x^4 \sec(e^x-1)\tan(e^x-1)\cdot e^x + \sec(e^x-1)\cdot 8x^3 + \frac{1}{\sqrt{1-(5x)^2}}\cdot 5$$

$$= 2x^4 e^x \sec(e^x-1)\tan(e^x-1) + 8x^3\sec(e^x-1) + \frac{5}{\sqrt{1-25x^2}}$$

7. $y = 6\csc(\ln x) - \sqrt[3]{4x+2}$ $= 6\csc(\ln x) - (4x+2)^{1/3}$

$$y' = 6\left(-\csc(\ln x)\cot(\ln x)\right)\cdot\frac{1}{x} - \frac{1}{3}(4x+2)^{-2/3}\cdot 4$$

$$= \frac{-6\csc(\ln x)\cot(\ln x)}{x} - \frac{4}{3(4x+2)^{2/3}}$$

8. $2x - 4x\ln y = 3\sin(xy) + 2$ (find $\frac{dy}{dx}$) implicit differentiation

product rule product rule

$$2 - \left[4x\cdot\frac{1}{y}\frac{dy}{dx} + \ln y\cdot 4\right] = 3\cos(xy)\left[x\frac{dy}{dx} + y\cdot 1\right]$$

$$2 - \frac{4x}{y}\frac{dy}{dx} - 4\ln y = 3x\cos(xy)\frac{dy}{dx} + 3y\cos(xy)$$

$$2 - 4\ln y - 3y\cos(xy) = 3x\cos(xy)\frac{dy}{dx} + \frac{4x}{y}\frac{dy}{dx}$$

$$2 - 4\ln y - 3y\cos(xy) = \frac{dy}{dx}\left(3x\cos(xy) + \frac{4x}{y}\right)$$

$$\frac{dy}{dx} = \frac{2 - 4\ln y - 3y\cos(xy)}{3x\cos(xy) + \frac{4x}{y}}\cdot\frac{y}{y} = \frac{y\left(2 - 4\ln y - 3y\cos(xy)\right)}{3xy\cos(xy) + 4x}$$

complex fraction