Technique for Solving Related Rates and Optimization Problems

	Related Rates (24.4)	Optimization (24.7)
Step 1*	Draw a picture and label variables • Don't assign variables to things that aren't changing. • Don't assign numbers to things that are changing.	Draw a picture and label variables
Step 2	Identify the 2 rates in the question: 1 is given, 1 is asked for Both will be $\frac{d(something)}{dt}$	Identify the quantity to be optimized
Step 3	Find an equation involving only the 2 variables in the rates from Step 2	Find an equation for the quantity to be optimized in terms of only 1 independent variable. Determine if the domain of this equation is open or closed.
Step 4	Take the derivative with respect to *t* of both sides – this is implicit differentiation	Take the derivative with respect to the independent variable – this is regular differentiation
Step 5	Evaluate at the numbers given in the question	Solve $derivative = 0$ • Use the FDT or SDT to verify that the critical value(s) give a max/min • If the domain is closed, the max/min could also occur at a closed endpoint – compare function values
Step 6	Include a unit	Find the quantity that the question asks for
Step 7		Include a unit

*If an equation is given in the question, you can skip this step

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The types of equations you should look for:

- For a question involving a triangle:
  $a^2 + b^2 = c^2$, similar triangles (could occur in cones), trig functions
- Geometric formulas:
  You should know:
  - ♦ Circle
    Area $A = \pi r^2$, circumference $C = 2\pi r$
  - ♦ Rectangle
    Area $A = lw$, perimeter $P = 2l + 2w$
  - ♦ Triangle
    Area $A = \frac{1}{2}bh$
  - ♦ Box
    Volume $V = lwh$, surface area of all 6 sides $A = 2lw + 2wh + 2lh$

    Special case: cube
    $$V = x^3, \qquad A = 6x^2$$
  - ♦ Cylinder
    Volume $V = \pi r^2 h$, surface area of all sides $A = 2(\pi r^2) + 2\pi rh$

  You would be given:

  - ♦ Volume and surface area of a sphere
  - ♦ Volume and surface area of a cone

- Distance formula
  $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$