Math 191 Practice Test Questions

1. Evaluate $\lim_{x \to -6} \dfrac{x^2 + 2x - 24}{x^2 + 11x + 30}$.

2. Let $f(x) = 2x + \dfrac{3}{x-1}$. Find $f'(x)$ using the limit definition.

3. Find the slope of the tangent line to $f(x) = x^6 - 3x^3 + 5x^2$ at the point $x = -2$.

4. An object's displacement (in metres) is given by $s = (t^3 + 2t + 2)(5t^2 + 6)$, where t is measured in seconds. Find the instantaneous rate of change of the object's displacement. Include the correct units, and simplify your answer.

5. Let $f(x) = 2x^4 + 7x^3 + x$. Find all the higher derivatives of $f(x)$.

6. Differentiate $f(x) = x^5(x+3)(x^3+2)$. Simplify your answer.

7. Let $y = \dfrac{x^2 - 21}{x + 5}$.

   (a) Find $\frac{dy}{dx}$.

   (b) For which x-values is the tangent line to y horizontal?

   (c) For which x-values is y differentiable?

8. Find $f'(-1)$ for $f(x) = [(2x+1)^4 + 6x]^{-2}$.

9. Let $f(x) = \sqrt{7x^4 + 1}(5 - 3x)$. Find and simplify $f'(x)$.

10. Find $\frac{dy}{dx}$ for $x^4 + y^3 - 6(x^2 + 3)y = 11x$.

11. Find the equation of the tangent line to $y = 2x^2 + 5x$ at the point where the slope of the tangent line is 13. Write your answer in the form $y = mx + b$.

12. Find the equations of the tangent line and normal line to the curve $y = \dfrac{1}{3x^2 + 4}$ at the point $(-1, \frac{1}{7})$. Write the lines in the form $ax + by + c = 0$.

13. We want to solve $x^3 - 5\sqrt{x} = 50$ using Newton's Method. Find x_2 using $x_1 = 4$. Round your answer to 2 decimal places.

14. An object's position (in metres) at t seconds is given by $x = t^3 - 4t^2, y = 1 + \frac{1}{4}t^2$. Find the object's velocity at $t = 2$ seconds.

15. A spherical balloon is being inflated so that the radius is increasing at a rate of 0.2 cm/s. At what rate is the volume changing at the moment when the volume is 2304π cm^3? Include units in your answer. *Hint:* $V = \frac{4}{3}\pi r^3$.

16. Let $f(x) = 8x^3 - 72x^2 - 648x$.

   (a) Find $f'(x)$.
   (b) Find $f''(x)$.
   (c) Find all relative maximum and relative minimum points. Indicate whether each point is a maximum or a minimum.
   (d) Find all points of inflection.
   (e) On which interval(s) is $f(x)$ concave up?
   (f) Sketch the curve with all points from parts (c) and (d) labelled.

17. A rectangular box is open at the top and has a square base. The volume of the box is $13,500$ cm^3. Find the dimensions of the box that minimize its surface area.

18. Approximate $\sqrt[4]{15.8}$ using a linear approximation. Express your final answer as a fraction.

19. Let A be the area of a circle with radius r. Find $\frac{dr}{r}$ given that $\frac{dA}{A} = 0.07$.

20. Find $\frac{dy}{dx}|_{x=1}$ for $y = \cos^3(7x-2) + \sec(5\sqrt{x}+1)$, rounding your answer to 2 decimal places.

21. Find $\frac{dy}{dx}$ for $\sin(xy) + \cos(2y) = x^2$.

22. Find $f'(0)$ for $f(x) = (2x+5)\cos^{-1} x + \frac{1}{3}\tan^{-1}(8x+1)$.

23. Find $f'(x)$ for $f(x) = \ln\sqrt{\frac{x^2+3}{5x-4}}$. Simplify your answer.

24. Find $f'(x)$ for $f(x) = 5^{4x^2-3x}$.

25. Find y' for $y = \dfrac{3(e^{2x} - e^{-2x})}{e^{2x}}$.

26. Find the angle θ between 0 and $\frac{\pi}{2}$ at which the function $f(\theta) = 4\sin\theta + 7\cos\theta$ is maximized. Leave your answer in exact form.

27. Find the equation of the line tangent to $y = \tan^{-1} 2x$ at $x = \frac{1}{2}$. Leave your answer in slope-intercept form.

28. Find the area of the largest rectangle that can be inscribed under the graph of $y = e^{-x}$ in the first quadrant.

29. Evaluate
$$\int_0^1 x^3(2x^4+1)^5\,dx$$

30. Evaluate
$$\int \sqrt{x}\,(x+2)\,dx$$

31. Evaluate

$$\int \frac{x^2}{(2x^3+1)^5}\,dx$$

32. Find y in terms of x if $\frac{dy}{dx} = \sqrt{6x-3}$ and the curve $y = f(x)$ passes through point $(2, -1)$.

33. (a) Approximate

$$\int_0^1 \sqrt{1+x^3}\,dx$$

by using the trapezoidal rule with $n = 5$.

(b) Use Simpson's rule to approximate

$$\int_{1.0}^{2.8} f(x)\,dx$$

using the following data points.

x	1.0	1.3	1.6	1.9	2.2	2.5	2.8
$f(x)$	3.2	4.1	5.2	4.6	4.2	5.1	5.7

34. An object starts from an initial displacement of 5 m, with an initial velocity of 10 m/s (at time $t = 0$). Determine its displacement s as a function of time t if its acceleration (in m/s^2) is given by

$$a = 12 - 0.6t$$

35. In coming to a stop, the acceleration of a car is $a = -5t$. If it is travelling at 40 m/s when the brakes are applied, how far does it travel while stopping ?

36. As a rocket burns, it consumes fuel and consequently gets lighter in mass. If a Saturn V rocket initially starts out with 2×10^6 kg of fuel and the rate of change of the mass of fuel (in kg/s) is given by

$$\frac{dm}{dt} = -t\sqrt{t^2+100},$$

how long does it take to burn all the fuel?

37. Find the area bounded by the parabola $y = x^2$, and the line $y = x + 2$.

38. Find the area bounded by the curves $y = x^3 - 3$, $x = 2$, $y = -1$, and $y = 3$.

39. Find the volume of the solid generated by revolving around the x-axis the first-quadrant area bounded by $y = 9 - x^2$, $x = 0$, and $y = 0$.

40. Use the shell method to find the volume of the solid generated by revolving around the y-axis the area bounded by $y = 3x^2 - x^3$ and $y = 0$.

41. Find the coordinates of the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$.

42. Find the coordinates of the centroid of the area bounded by $y = x^3$, $y = 0$, and $x = 2$.

43. Find the surface area of the solid obtained by revolving the graph of $y = x^3$ from $x = 1$ to $x = 2$ around the x-axis.

44. Find the arc length of the curve $y = 1 + 2x^{3/2}$ over the interval $0 \leq x \leq 1$.

45. The electric current (in A) as a function of time t (in s) for a certain circuit is given by

$$i = 8t - t^2.$$

Find the average current for the first 4 seconds, i.e. over $0 \leq t \leq 4$.

46. Find AB and BA (if they are defined).

(a) $A = \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix}$

47. Find the inverse of the following 2×2 matrices (if it exists) using the formula.

(a) $A = \begin{bmatrix} 1 & -6 \\ 4 & -7 \end{bmatrix}$

(b) $B = \begin{bmatrix} 12 & -3 \\ -8 & 2 \end{bmatrix}$

48. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ if it exists.

49. Solve the systems below by finding A^{-1}:

(a) $\begin{cases} 8x - y = 17 \\ -4x + y = -5 \end{cases}$

(b) $\begin{cases} 4x + 4z = -4 \\ x + y + 2z = -4 \\ x + y + z = -2 \end{cases}$

50. Solve the following systems using Gauss-Jordan Elimination. If there are infinitely many solutions, give two particular solutions.

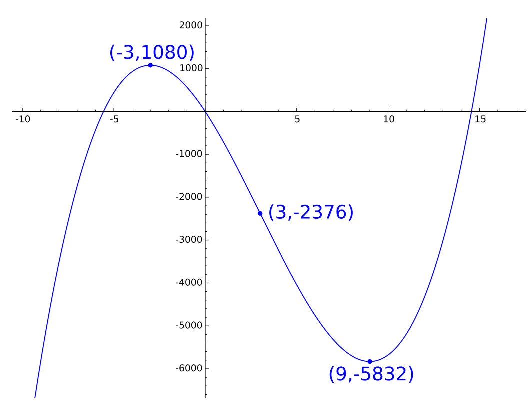
(a) $\begin{cases} x + 3y - 2z = 9 \\ 2x - y + 4z = 6 \\ -3x + 2y - 3z = -1 \end{cases}$

(b) $\begin{cases} 3x - 18y + 21z = 12 \\ 2x + 7y - 6z = 3 \end{cases}$

(c) $\begin{cases} x + 3y + 3z = 12 \\ 2x + 20y + 10z = 8 \\ x + 10y + 5z = 0 \end{cases}$

Answers:

1. 10
2. $2 - \dfrac{3}{(x-1)^2}$
3. -248
4. $s'(t) = 25t^4 + 48t^2 + 20t + 12$ m/s
5. $f'(x) = 8x^3 + 21x^2 + 1$,
   $f''(x) = 24x^2 + 42x$,
   $f'''(x) = 48x + 42$,
   $f^{(4)} = 48$,
   $f^{(n)} = 0$ for $n \geq 5$
6. $9x^8 + 24x^7 + 12x^5 + 30x^4$
7. (a) $\dfrac{x^2 + 10x + 21}{(x+5)^2}$
   (b) $-3, -7$
   (c) $x \neq -5$
8. $\dfrac{-4}{125}$
9. $\dfrac{-63x^4 + 70x^3 - 3}{\sqrt{7x^4 + 1}}$
10. $\dfrac{11 - 4x^3 + 12xy}{3y^2 - 6x^2 - 18}$
11. $y = 13x - 8$
12. tangent $6x - 49y + 13 = 0$,
    normal $343x + 42y + 337 = 0$
13. 3.91
14. 4.1 m/s at 166.0°
15. 115.2π cm^3/s
16. (a) $24x^2 - 144x - 648$
    (b) $48x - 144$
    (c) relative maximum at $(-3, 1080)$,
    relative minimum at $(9, -5832)$
    (d) $(3, -2376)$
    (e) $(3, \infty)$
    (f)



17. 30 cm x 30 cm x 15 cm
18. $\dfrac{319}{160}$

19. 0.035

20. 0.86

21. $\dfrac{2x - y\cos(xy)}{x\cos(xy) - 2\sin(2y)}$

22. $\pi - \dfrac{11}{3}$

23. $\dfrac{5x^2 - 8x - 15}{(x^2+3)(10x-8)}$

24. $(\ln 5)(8x-3)5^{4x^2-3x}$

25. $12e^{-4x}$

26. $\tan^{-1}\left(\frac{4}{7}\right)$

27. $y = x + \dfrac{\pi - 2}{4}$

28. $\dfrac{1}{e}$

29. $\dfrac{91}{6}$

30. $\frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + C$

31. $\dfrac{-1}{24(2x^3+1)^4} + C$

32. $y = \frac{1}{9}(6x-3)^{3/2} - 4$

33. (a) 1.115
    (b) 8.29

34. $s = 6t^2 - \frac{1}{10}t^3 + 10t + 5$

35. 106.7 m

36. 181.4 s

37. $A = \dfrac{9}{2}$

38. $A \simeq 1.7128$

39. $V = \frac{648\pi}{5} \simeq 407.15$

40. $V = \frac{243\pi}{10} \simeq 76.34$

41. $(\overline{x}, \overline{y}) = \left(\frac{1}{2}, \frac{2}{5}\right)$

42. $(\overline{x}, \overline{y}) = \left(\frac{8}{5}, \frac{16}{7}\right)$

43. $SA \simeq 199.48$

44. $s \simeq 2.268$

45. $i_{\text{av}} \simeq 10.67$ A

46. (a) $AB = \begin{bmatrix} -14 & -18 \\ -22 & -26 \end{bmatrix}$,

    $BA = \begin{bmatrix} 2 & -4 \\ 13 & -42 \end{bmatrix}$

    (b) $AB = \begin{bmatrix} 7 & 3 \\ 4 & 8 \end{bmatrix}$,

    $BA = \begin{bmatrix} 18 & -13 & 11 \\ 25 & -19 & 20 \\ 13 & -11 & 16 \end{bmatrix}$

    (c) $AB = \begin{bmatrix} -20 \\ -27 \end{bmatrix}$, BA is undefined

47. (a) $\dfrac{1}{17}\begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$

    (b) B^{-1} does not exist

48. $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1/2 & -1/4 \\ -1 & -1/2 & 5/4 \end{bmatrix}$

49. (a) $x = 3,\ y = 7$

    (b) $x = 1,\ y = -1,\ z = -2$

50. (a) $x = 1,\ y = 4,\ z = 2$

    (b) $x = \frac{46}{19} - \frac{13}{19}t,\ y = -\frac{5}{19} + \frac{20}{19}t,\ z = t$

    $x = \frac{46}{19},\ y = -\frac{5}{19},\ z = 0;$

    $x = \frac{33}{19},\ y = \frac{15}{19},\ z = 1$

    (c) no solution