Practice Test Questions 11-20

11. $y = 2x^2 + 5x$

$y' = 4x + 5$ solve for $x$ when $y' = 13$

$$4x + 5 = 13$$
$$4x = 8$$
$$x = 2$$

when $x = 2$, $y = 2(2)^2 + 5(2) = 18$

The tangent slope is 13 at the point $(2, 18)$

$$y = mx + b$$
$$18 = 13(2) + b \quad \text{solve for } b$$
$$b = -8$$

so $y = 13x - 8$

12. $y = \dfrac{1}{3x^2+4} = (3x^2+4)^{-1}$

$$y' = (-1)(3x^2+4)^{-2}(6x) = -\frac{6x}{(3x^2+4)^2}$$

$$y'\Big|_{x=-1} = \frac{-6(-1)}{(3(-1)^2+4)^2} = \frac{6}{49}$$

tangent: $m = \dfrac{6}{49}$ so

$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{7} = \frac{6}{49}(x+1)$$
$$y - \frac{1}{7} = \frac{6}{49}x + \frac{6}{49}$$
$$49y - 7 = 6x + 6$$
$$6x - 49y + 13 = 0$$

normal: $m = -\frac{49}{6}$ so

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{7} = -\frac{49}{6}(x+1)$$

$$y - \frac{1}{7} = -\frac{49}{6}x - \frac{49}{6}$$

$$42y - 6 = -343x - 343$$

$$343x + 42y + 337 = 0$$

13. $x^3 - 5\sqrt{x} = 50$

$$\underbrace{x^3 - 5x^{1/2} - 50}_{f(x)} = 0$$

$$f'(x) = 3x^2 - \frac{5}{2}x^{-1/2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4 - \frac{4^3 - 5\sqrt{4} - 50}{3(4)^2 - \frac{5}{2\sqrt{4}}} \approx 3.91$$

14. $x = t^3 - 4t^2$

$v_x = 3t^2 - 8t$

$v_x\big|_{t=2} = 3(2)^2 - 8(2)$

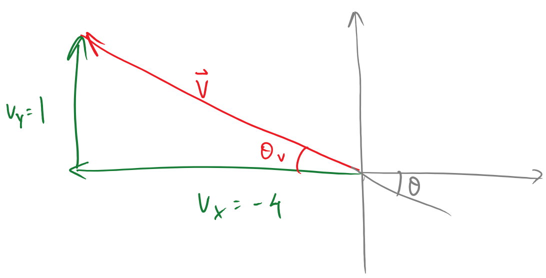
$= -4$

$y = 1 + \frac{1}{4}t^2$

$v_y = \frac{1}{2}t$

$v_y\big|_{t=2} = \frac{1}{2}(2)$

$= 1$




$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(-4)^2 + 1^2} = \sqrt{17} \approx 4.12$$

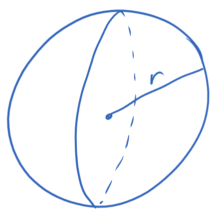
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{1}{-4}\right) = -14.04^\circ$$

but $\theta_v$ is in QII

so $\theta_v = \theta + 180^\circ = 165.96^\circ$

$$\vec{v} = 4.12 \text{ m/s} \ @ \ 165.96^\circ$$

15.




given rate: $\frac{dr}{dt} = 0.2 \ \frac{cm}{s}$

wanted rate: $\left.\frac{dV}{dt}\right|_{V=2304\pi}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

we need r when $V = 2304\pi$

$$2304\cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$$

$$r = 12$$

$$\left.\frac{dV}{dt}\right|_{r=12} = 4\pi(12)^2(0.2)$$

$$= 115.2\pi \approx 361.91 \ \frac{cm^3}{s}$$

16. $f(x) = 8x^3 - 72x^2 - 648x$

a) $f'(x) = 24x^2 - 144x - 648 = 24(x^2 - 6x - 27)$

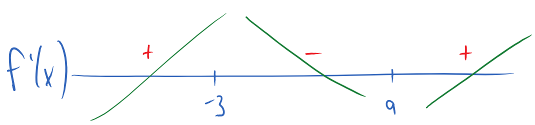
b) $f''(x) = 24(2x - 6) = 48(x-3)$

c) rel. max./min occur when $f'(x) = 0$

$$24(x^2 - 6x - 27) = 0$$

$$24(x-9)(x+3) = 0$$

$$x = 9, -3$$




$$f'(-4) = \underset{+}{24}\underset{-}{(-4-9)}\underset{-}{(-4+3)}$$

rel. max. at $x = -3$ $\qquad f(-3) = 8(-3)^3 - 72(-3)^2 - 648(-3) = 1080$

$(-3, 1080)$

rel. min. at $x = 9$ $\qquad f(9) = 8(9)^3 - 72(9)^2 - 648(9) = -5832$

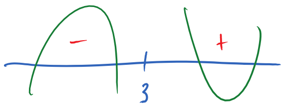
$(9, -5832)$

d) points of inflection occur when $f''(x) = 0$

$$48(x-3) = 0$$

$$x = 3$$

$f''(x)$




$$f''(0) = \underset{+}{48}(\underset{-}{0-3})$$

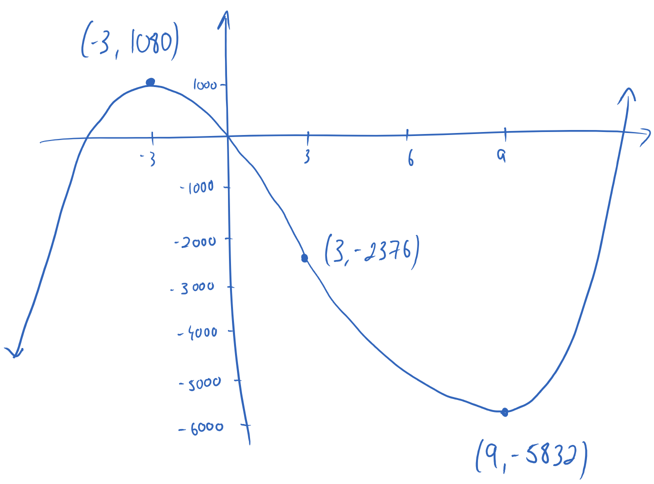
p.o.i. at $x = 3$ $\qquad f(3) = 8(3)^3 - 72(3)^2 - 648(3) = -2376$

$$(3, -2376)$$

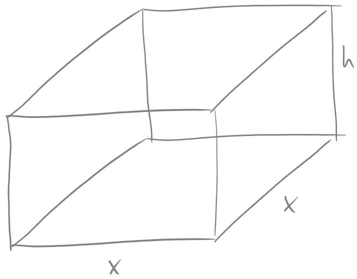
e) from part d) we see $f(x)$ is concave up on

$$(3, \infty)$$

f)



17.




we want to minimize

$A = x^2 + 4xh$

↑ bottom ↑ 4 sides

we should only have 1 variable

$V = x^2 h$

$13500 = x^2 h$

$h = \dfrac{13500}{x^2}$

$A = x^2 + 4x \cdot \dfrac{13500}{x^2} = x^2 + 54000x^{-1}$ , domain $x > 0$

if a min A exists, it occurs when $A' = 0$

$A' = 2x - 54000x^{-2} = 2x - \dfrac{54000}{x^2} = \dfrac{2x^3 - 54000}{x^2}$

solve $A' = 0$:

$2x^3 - 54000 = 0$

$2x^3 = 54000$

$x^3 = 27000$

$x = 30$ ← verify min

SDT: $A'' = 2 + 108x^{-3}$

$A''(30) = 2 + \dfrac{180}{30^3} > 0$ ∪ min ✓

Question: dimensions

when $x = 30$, $h = \dfrac{13500}{30^2} = 15$

dimensions: 30 cm × 30 cm × 15 cm

18. $\sqrt[4]{15.8}$

linear approximation: use a tangent line nearby

15.8 is close to 16 and we know the exact value of $\sqrt[4]{16}$
so we'll use the tangent line to $f(x) = \sqrt[4]{x}$ at $x = 16$

$$y - y_1 = m(x - x_1)$$
$$\quad ? \quad\ ? \qquad 16$$

$$y_1 = f(x_1) = f(16) = \sqrt[4]{16} = 2$$

$$m = f'(x_1) = f'(16) = \frac{1}{4(16)^{3/4}} = \frac{1}{32}$$

$$f(x) = \sqrt[4]{x} = x^{1/4}$$
$$f'(x) = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$$

so $y - 2 = \frac{1}{32}(x - 16)$

$$y = \frac{1}{32}x - \frac{1}{2} + 2 = \frac{1}{32}x + \frac{3}{2}$$

$$f(x) \approx y = \frac{1}{32}x + \frac{3}{2} \quad \text{for} \quad x \approx 16$$

$$f(15.8) = \sqrt[4]{15.8} \approx \frac{1}{32}(15.8) + \frac{3}{2} = \frac{319}{160}$$

19. $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r\, dr$$

$$\frac{dA}{A} = \frac{2\pi r\, dr}{\pi r^2} = 2\frac{dr}{r}$$

$$0.07 = 2\frac{dr}{r}$$

$$\frac{dr}{r} = 0.035$$

20. $y = \cos^3(7x-2) + \sec(5\sqrt{x}+1)$

$= [\cos(7x-2)]^3 + \sec(5x^{1/2}+1)$

$\frac{dy}{dx} = 3[\cos(7x-2)]^2[-\sin(7x-2)](7) + \sec(5x^{1/2}+1)\tan(5x^{1/2}+1)\cdot\left(\frac{5}{2}x^{-1/2}\right)$

$\left.\frac{dy}{dx}\right|_{x=1} = -21\cos^2(7\cdot1-2)\sin(7\cdot1-2) + \frac{5}{2\sqrt{1}}\sec(5\cdot1^{1/2}+1)\tan(5\cdot1^{1/2}+1)$

$\uparrow$
radians

$= 0.86$