

Practice Test Questions 1-10

$$1. \lim_{x \rightarrow -6} \frac{x^2 + 2x - 24}{x^2 + 11x + 30} = \lim_{x \rightarrow -6} \frac{\cancel{(x+6)}(x-4)}{\cancel{(x+6)}(x+5)} = \frac{-6-4}{-6+5} = \frac{-10}{-1} = 10$$

$$2. f(x) = 2x + \frac{3}{x-1}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x+h) + \frac{3}{(x+h)-1} - \left[2x + \frac{3}{x-1} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x} + 2h - \cancel{2x} + \frac{3}{(x+h-1)(x-1)} - \frac{3}{(x-1)(x+h-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h + \frac{\cancel{3x} - \cancel{3} - \cancel{3x} - \cancel{3h} + \cancel{3}}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2h}}{\cancel{h}} - \frac{\cancel{3h}}{h(x+h-1)(x-1)}$$

$$= 2 - \frac{3}{(x+0-1)(x-1)}$$

$$= 2 - \frac{3}{(x-1)^2}$$

$$3. f(x) = x^6 - 3x^3 + 5x^2$$

$$m = f'(-2)$$

$$f'(x) = 6x^5 - 9x^2 + 10x$$

$$f'(-2) = 6(-2)^5 - 9(-2)^2 + 10(-2) = -248$$

$$4. \quad s = \underbrace{(t^3 + 2t + 2)}_u \cdot \underbrace{(5t^2 + 6)}_v$$

$$\begin{aligned} s'(t) &= uv' + vu' = (t^3 + 2t + 2)(10t) + (3t^2 + 2)(5t^2 + 6) \\ &= 10t^4 + 20t^2 + 20t + 15t^4 + 18t^2 + 10t^2 + 12 \\ &= 25t^4 + 48t^2 + 20t + 12 \quad \text{m/s} \end{aligned}$$

alternatively: expand the product in s first, then take the derivative

$$5. \quad f(x) = 2x^4 + 7x^3 + x$$

$$f'(x) = 8x^3 + 21x^2 + 1$$

$$f''(x) = 24x^2 + 42x$$

$$f'''(x) = 48x + 42$$

$$f^{(4)}(x) = 48$$

$$f^{(n)}(x) = 0 \quad \text{for } n \geq 5$$

$$6. \quad f(x) = x^5(x+3)(x^3+2) = (x^6 + 3x^5)(x^3+2) = x^9 + 2x^6 + 3x^8 + 6x^5$$

$$f'(x) = 9x^8 + 24x^7 + 12x^5 + 30x^4$$

$$\text{OR } 3x^4(3x^4 + 8x^3 + 4x + 10)$$

alternatively: use the product rule and then expand

$$9. f(x) = \sqrt{7x^4+1} (5-3x) = \underbrace{(7x^4+1)^{1/2}}_u \underbrace{(5-3x)}_v$$

$$\begin{aligned} f'(x) &= uv' + vu' = (7x^4+1)^{1/2} (-3) + (5-3x) \left(\frac{1}{2}\right) (7x^4+1)^{-1/2} (28x^3) \\ &= -3(7x^4+1)^{1/2} \cdot \frac{(7x^4+1)^{1/2}}{(7x^4+1)^{1/2}} + \frac{70x^3 - 42x^4}{(7x^4+1)^{1/2}} \\ &= \frac{-3(7x^4+1) + 70x^3 - 42x^4}{(7x^4+1)^{1/2}} \\ &= \frac{-63x^4 + 70x^3 - 3}{\sqrt{7x^4+1}} \end{aligned}$$

$$10. x^4 + y^3 - 6(x^2+3)y = 11x$$

$$x^4 + y^3 - \underbrace{6x^2y}_{\text{product rule}} - 18y = 11x$$

$$4x^3 + 3y^2 \frac{dy}{dx} - \left[6x^2 \frac{dy}{dx} + 12xy \right] - 18 \frac{dy}{dx} = 11$$

$$3y^2 \frac{dy}{dx} - 6x^2 \frac{dy}{dx} - 18 \frac{dy}{dx} = 11 - 4x^3 + 12xy$$

$$(3y^2 - 6x^2 - 18) \frac{dy}{dx} = 11 - 4x^3 + 12xy$$

$$\frac{dy}{dx} = \frac{11 - 4x^3 + 12xy}{3y^2 - 6x^2 - 18}$$