Practice Test Questions 21-30

21. $\sin(xy) + \cos(2y) = x^2$

$$\cos(xy)\left[x\frac{dy}{dx} + y(1)\right] - \sin(2y)\cdot 2\frac{dy}{dx} = 2x$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) - 2\sin(2y)\frac{dy}{dx} = 2x$$

$$x\cos(xy)\frac{dy}{dx} - 2\sin(2y)\frac{dy}{dx} = 2x - y\cos(xy)$$

$$\left[x\cos(xy) - 2\sin(2y)\right]\frac{dy}{dx} = 2x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy) - 2\sin(2y)}$$

22. $f(x) = \underbrace{(2x+5)\cos^{-1}x}_{\text{product rule}} + \frac{1}{3}\tan^{-1}(8x+1)$

$$f'(x) = (2x+5)\cdot\frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x\,(2) + \frac{1}{3}\cdot\frac{1}{1+(8x+1)^2}\cdot 8$$

$$f'(0) = \frac{-(2\cdot 0+5)}{\sqrt{1-0^2}} + 2\cos^{-1}0 + \frac{8}{3[1+(8\cdot 0+1)^2]}$$

$$= -5 + 2\left(\frac{\pi}{2}\right) + \frac{4}{3}$$

$$= \pi - \frac{11}{3}$$

23. $f(x) = \ln\sqrt{\dfrac{x^2+3}{5x-4}} = \ln\left(\dfrac{x^2+3}{5x-4}\right)^{1/2} = \dfrac{1}{2}\ln\left(\dfrac{x^2+3}{5x-4}\right) = \dfrac{1}{2}\left[\ln(x^2+3) - \ln(5x-4)\right]$

$$f'(x) = \frac{1}{2}\left[\frac{1}{x^2+3}\cdot 2x - \frac{1}{5x-4}\cdot 5\right]$$

$$= \frac{1}{2}\left[\frac{2x(5x-4) - 5(x^2+3)}{(x^2+3)(5x-4)}\right]$$

$$= \frac{5x^2 - 8x - 15}{2(x^2+3)(5x-4)}$$

Alternatively, use the chain rule: $f'(x) = \left(\dfrac{5x-4}{x^2+3}\right)^{1/2}\dfrac{1}{2}\left(\dfrac{x^2+3}{5x-4}\right)^{-1/2}\left[\dfrac{(5x-4)(2x) - (x^2+3)(5)}{(5x-4)^2}\right]$

24. $f(x) = 5^{4x^2 - 3x}$

$$f'(x) = 5^{4x^2-3x}(\ln 5)(8x-3)$$

25. $y = \dfrac{3(e^{2x} - e^{-2x})}{e^{2x}} = 3\left[\dfrac{e^{2x}}{e^{2x}} - \dfrac{e^{-2x}}{e^{2x}}\right] = 3 - 3e^{-4x}$

$$y' = -3e^{-4x}(-4) = \frac{12}{e^{4x}}$$

26. $f(\theta) = 4\sin\theta + 7\cos\theta$

$$f'(\theta) = 4\cos\theta - 7\sin\theta$$

to maximize f, solve $f'(\theta) = 0$

$$4\cos\theta - 7\sin\theta = 0$$

$$4\cos\theta = 7\sin\theta$$

$$\frac{4}{7} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{4}{7}\right)$$ ← this is the only solution on $(0, \frac{\pi}{2})$ so it must be a max

27. $y = \tan^{-1} 2x$

$$y' = \frac{1}{1+(2x)^2}(2) = \frac{2}{1+4x^2}$$

$$m = y'\Big|_{x=\frac{1}{2}} = \frac{2}{1+4(\frac{1}{2})^2} = \frac{2}{2} = 1$$

$$y_1 = y\Big|_{x=\frac{1}{2}} = \tan^{-1} 2\left(\tfrac{1}{2}\right)$$

$$= \tan^{-1} 1 = 45° = \frac{\pi}{4}$$

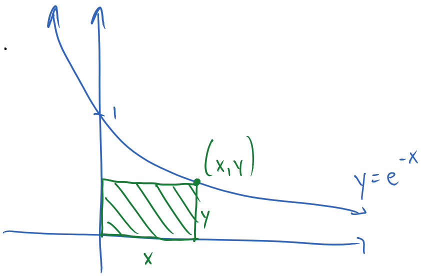
$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{4} = 1\left(x - \frac{1}{2}\right)$$

$$y = x - \frac{1}{2} + \frac{\pi}{4}$$

$$y = x + \frac{\pi - 2}{4}$$

28.



we want to maximize area

$$A = xy = xe^{-x}$$

$$A' = xe^{-x}(-1) + e^{-x}(1)$$
$$= e^{-x}(-x+1)$$

the max occurs when $A' = 0$

$$e^{-x}(-x+1) = 0$$
$$x = 1$$

when $x=1$, $A = 1e^{-1} = \frac{1}{e}$

29. $\int_0^1 x^3(2x^4+1)^5\,dx$

let $u = 2x^4+1$
$du = 8x^3\,dx$
$\frac{1}{8}du = x^3\,dx$

$$= \frac{1}{8}\int_{x=0}^{x=1} u^5\,du$$

$$= \frac{1}{8}\cdot\frac{1}{6}u^6\Big|_{x=0}^{x=1}$$

$$= \frac{1}{48}(2x^4+1)^6\Big|_0^1$$

$$= \frac{1}{48}\left[(2\cdot 1^4+1)^6 - (2\cdot 0^4+1)^6\right]$$

$$= \frac{91}{6}$$

30. $\int \sqrt{x}(x+2)\,dx$

$$= \int x^{1/2}(x+2)\,dx$$

$$= \int\left(x^{3/2} + 2x^{1/2}\right)dx$$

$$= \frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + C$$