Practice Test Questions 31-40

31. $\int \frac{x^2}{(2x^3+1)^5}\,dx$

let $u = 2x^3+1$
$du = 6x^2\,dx$
$\frac{1}{6}\,du = x^2\,dx$

$= \frac{1}{6}\int u^{-5}\,du$

$= \frac{1}{6}\left(\frac{1}{-4}\right)u^{-4} + C$

$= -\frac{1}{24(2x^3+1)^4} + C$

32. $\frac{dy}{dx} = \sqrt{6x-3}$

$y = \int \sqrt{6x-3}\,dx$

$= \int (\underbrace{6x-3}_{\text{linear}})^{1/2}\,dx$

instead of using the shortcut for linear functions, you can also use $u = 6x-3$
⋮

$= \frac{1}{6}\cdot\frac{2}{3}(6x-3)^{3/2} + C$

$y = \frac{1}{9}(6x-3)^{3/2} + C$

now use $(2,-1)$ to solve for C

$-1 = \frac{1}{9}(6\cdot 2-3)^{3/2} + C$

$-1 = 3 + C$

$C = -4$

so $y = \frac{1}{9}(6x-3)^{3/2} - 4$

33. a)

0	0.2	0.4	0.6	0.8	1
x_0	x_1	x_2	x_3	x_4	x_5

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{5} = 0.2$$

$$\int_0^1 \sqrt{1+x^3}\,dx \approx \frac{1-0}{2(5)}\left[\sqrt{1+0^3} + 2\sqrt{1+0.2^3} + 2\sqrt{1+0.4^3} + 2\sqrt{1+0.6^3} + 2\sqrt{1+0.8^3} + \sqrt{1+1^3}\right]$$

$$\approx 1.11499$$

b) $$\int_{1.0}^{2.8} f(x)\,dx \approx \frac{2.8-1}{3(6)}\left[3.2 + 4(4.1) + 2(5.2) + 4(4.6) + 2(4.2) + 4(5.1) + 5.7\right]$$

$$= 8.29$$

34.

$$v = \int a\,dt$$

$$= \int (12 - 0.6t)\,dt$$

$$v = 12t - 0.3t^2 + C_1$$

solve for C_1 using $v = 10$ at $t = 0$

$$10 = 12(0) - 0.3(0)^2 + C_1$$

$$C_1 = 10$$

so $v = 12t - 0.3t^2 + 10$

$$s = \int v\,dt$$

$$= \int (12t - 0.3t^2 + 10)\,dt$$

$$s = 6t^2 - 0.1t^3 + 10t + C_2$$

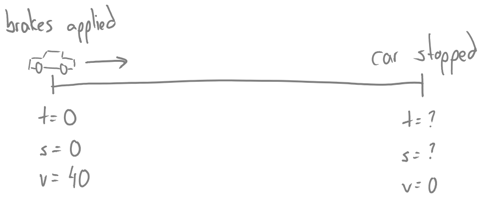
solve for C_2 using $s = 5$ when $t = 0$

$$5 = 6(0)^2 - 0.1(0)^3 + 10(0) + C_2$$

$$C_2 = 5$$

so $s = 6t^2 - 0.1t^3 + 10t + 5$ m

35. $a = -5t$



$$v = \int a\,dt = \int -5t\,dt$$

$$v = -\frac{5}{2}t^2 + C_1$$

use $v = 40$ at $t = 0$ to find C_1

$$40 = -\frac{5}{2}(0)^2 + C_1$$
$$C_1 = 40$$

$$v = -\frac{5}{2}t^2 + 40$$

we need t when $v = 0$

$$0 = -\frac{5}{2}t^2 + 40$$

$$\frac{5}{2}t^2 = 40$$
$$t^2 = 16$$
$$t = \pm 4 \quad \text{but } t > 0$$
$$\text{so } t = 4$$

we want $s(4) - s(0) = s(4)$ $\quad (s(0) = 0)$

$$s = \int v\,dt = \int \left(-\frac{5}{2}t^2 + 40\right)dt$$

$$s = -\frac{5}{6}t^3 + 40t + C_2$$

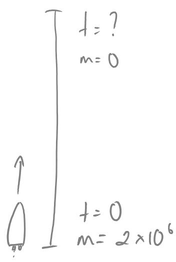
use $s = 0$ at $t = 0$ to find C_2

$$0 = -\frac{5}{6}(0)^3 + 40(0) + C_2$$
$$C_2 = 0$$

$$s = -\frac{5}{6}t^3 + 40t$$

$$s(4) = -\frac{5}{6}(4)^3 + 40(4) = \frac{320}{3} \approx 106.7 \text{ m}$$

36. $\dfrac{dm}{dt} = -t\sqrt{t^2+100}$



$m = \int -t\sqrt{t^2+100}\,dt$ let $u = t^2+100$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$= -\frac{1}{2}\int u^{1/2}\,du$

$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$m = -\frac{1}{3}(t^2+100)^{3/2} + C$

solve for C using $m = 2\times10^6$ when $t=0$

$2\times10^6 = -\frac{1}{3}(0^2+100)^{3/2} + C$

$C = 2\times10^6 + \frac{1000}{3}$

so $m = -\frac{1}{3}(t^2+100)^{3/2} + 2\times10^6 + \frac{1000}{3}$

we want t when $m=0$

$0 = -\frac{1}{3}(t^2+100)^{3/2} + 2\times10^6 + \frac{1000}{3}$

$0 = -(t^2+100)^{3/2} + 6001000$

$(t^2+100)^{3/2} = 6001000$

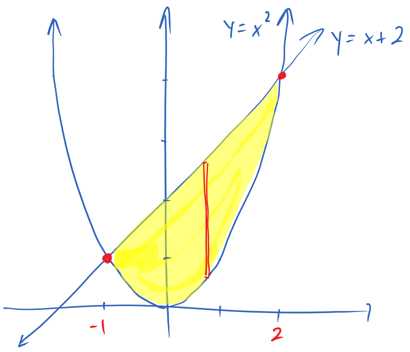
$t^2+100 = 6001000^{2/3}$

$t^2 = 6001000^{2/3} - 100$, $t>0$

$t = \sqrt{6001000^{2/3} - 100}$

≈ 181.45 s

37.



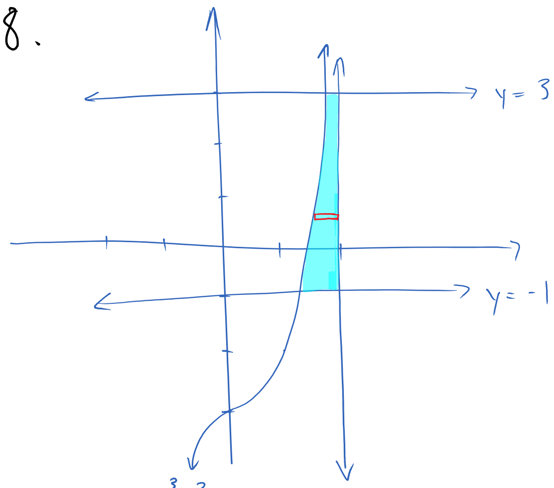
intersection points:

$$y=y$$
$$x^2=x+2$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2,-1$$

$$dA=(\text{top}-\text{bottom})\,dx$$
$$=(x+2-x^2)\,dx$$

$$A=\int_A dA=\int_{-1}^{2}(x+2-x^2)\,dx$$
$$=\frac{1}{2}x^2+2x-\frac{1}{3}x^3\Big|_{-1}^{2}$$
$$=\frac{9}{2}$$

38.

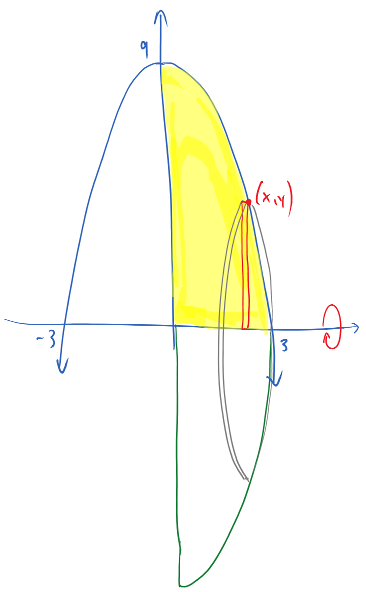


$$y=x^3-3$$
$$y+3=x^3$$
$$x=(y+3)^{1/3}$$

$$dA=(\text{right}-\text{left})\,dy$$
$$=\left(2-(y+3)^{1/3}\right)dy$$

$$A=\int_A dA$$
$$=\int_{-1}^{3}\Big(2-(\underbrace{y+3}_{\text{linear}})^{1/3}\Big)dy$$
$$=2y-\frac{3}{4}(y+3)^{4/3}\Big|_{-1}^{3}$$
$$\approx 1.7128$$

39.

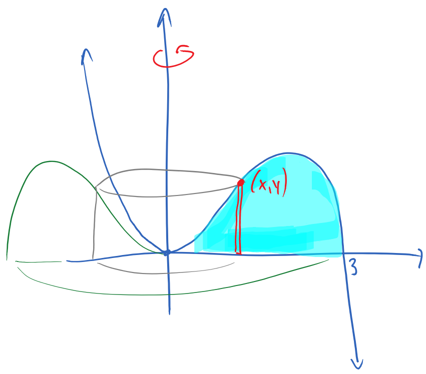


disk:
$$dV=\pi r^2\,dt$$
$$=\pi y^2\,dx$$
$$=\pi(9-x^2)^2\,dx$$
$$=\pi(81-18x^2+x^4)\,dx$$

$$V=\int_V dV$$
$$=\pi\int_0^3(81-18x^2+x^4)\,dx$$
$$=\frac{648\pi}{5}$$

40.

$$y = 3x^2 - x^3$$
$$= x^2(3-x)$$



shell: $dV = 2\pi r h dt$

$$= 2\pi x\, y\, dx$$
$$= 2\pi x (3x^2 - x^3)\, dx$$
$$= 2\pi (3x^3 - x^4)\, dx$$

$$V = \int_V dV$$
$$= 2\pi \int_0^3 (3x^3 - x^4)\, dx$$
$$= 2\pi \left(\frac{3}{4}x^4 - \frac{1}{5}x^5 \right) \Big|_0^3$$
$$= \frac{243\pi}{10}$$