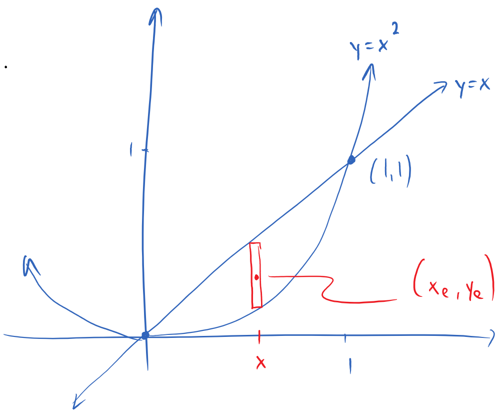
Practice Test Questions 41-50

41.



$$\bar{x} = \frac{1}{A}\int_A x_e \, dA$$

$$dA = (\text{top} - \text{bottom})\, dx$$

$$dA = (x - x^2)\, dx$$

$$x_e = x$$

$$A = \int_A dA$$

$$= \int_0^1 (x - x^2)\, dx$$

$$= \left(\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right)\Big|_0^1$$

$$A = \frac{1}{6}$$

$$y_e = \text{midpoint}$$

$$= \frac{\text{top} + \text{bottom}}{2}$$

$$y_e = \frac{x + x^2}{2}$$

$$\bar{x} = \frac{1}{A}\int_A x_e \, dA$$

$$= \frac{6}{1}\int_0^1 x(x - x^2)\, dx$$

$$= 6\int_0^1 (x^2 - x^3)\, dx$$

$$= 6\left(\tfrac{1}{3}x^3 - \tfrac{1}{4}x^4\right)\Big|_0^1$$

$$= 6\left(\tfrac{1}{12}\right)$$

$$= \frac{1}{2}$$

$$\bar{y} = \frac{1}{A}\int_A y_e \, dA$$

$$= \frac{6}{1}\int_0^1 \left(\frac{x + x^2}{2}\right)(x - x^2)\, dx$$

$$= 3\int_0^1 (x^2 - x^4)\, dx$$

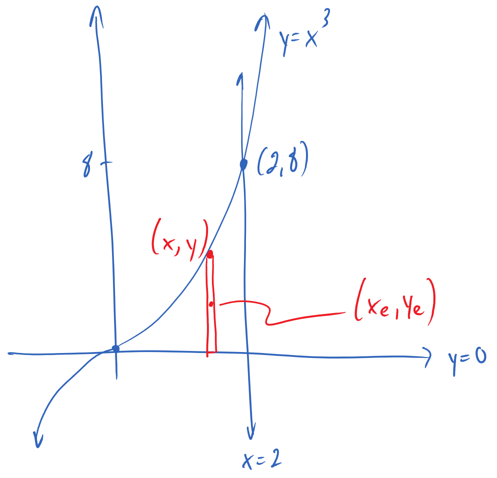
$$= 3\left(\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right)\Big|_0^1$$

$$= 3\left(\tfrac{2}{15}\right)$$

$$= \frac{2}{5}$$

$$(\bar{x}, \bar{y}) = \left(\tfrac{1}{2}, \tfrac{2}{5}\right)$$

42.



$$\bar{x} = \frac{1}{A}\int_A x_e \, dA$$

$$dA = y\,dx$$
$$dA = x^3 dx$$

$$A = \int_A dA = \int_0^2 x^3 dx = \frac{1}{4}x^4 \Big|_0^2$$
$$A = 4$$

$$x_e = x$$
$$y_e = \frac{1}{2}y = \frac{1}{2}x^3$$

$$\bar{x} = \frac{1}{A}\int_A x_e \, dA = \frac{1}{4}\int_0^2 x \, x^3 dx = \frac{1}{4}\int_0^2 x^4 dx = \frac{1}{4}\cdot\frac{1}{5}x^5\Big|_0^2 = \frac{8}{5}$$

$$\bar{y} = \frac{1}{A}\int_A y_e \, dA = \frac{1}{4}\int_0^2 \frac{1}{2}x^3 \, x^3 dx = \frac{1}{8}\int_0^2 x^6 dx = \frac{1}{8}\cdot\frac{1}{7}x^7\Big|_0^2 = \frac{16}{7}$$

$$(\bar{x}, \bar{y}) = \left(\frac{8}{5}, \frac{16}{7}\right)$$

43. $SA = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$\left(\frac{dy}{dx}\right)^2 = (3x^2)^2 = 9x^4$

$SA = 2\pi \int_1^2 x^3 \sqrt{1 + 9x^4}\, dx$

let $u = 1 + 9x^4$

$du = 36x^3\, dx$

$\frac{1}{36}\, du = x^3\, dx$

$= \frac{2\pi}{36} \int_{x=1}^{x=2} u^{1/2}\, du$

$= \frac{\pi}{18} \cdot \frac{2}{3} u^{3/2} \Big|_{x=1}^{x=2}$

$= \frac{\pi}{27} (1 + 9x^4)^{3/2} \Big|_1^2 \approx 199.48$

44. $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$y = 1 + 2x^{3/2}$

$\frac{dy}{dx} = 3x^{1/2}$

$\left(\frac{dy}{dx}\right)^2 = (3x^{1/2})^2 = 9x$

$s = \int_0^1 \sqrt{1 + 9x}\, dx$

$= \int_0^1 (\underbrace{1 + 9x}_{\text{linear}})^{1/2}\, dx$

$= \frac{1}{9} \cdot \frac{2}{3} (1 + 9x)^{3/2} \Big|_0^1 \approx 2.27$

45. $$y_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$i_{av} = \frac{1}{4-0}\int_0^4 \left(8t - t^2\right) dt$$

$$= \frac{1}{4}\left(4t^2 - \frac{1}{3}t^3\right)\Big|_0^4 = \frac{1}{4}\left(4\cdot 4^2 - \frac{1}{3}\cdot 4^3 - 0\right) = \frac{32}{3} \approx 10.67 \text{ A}$$

46. a) $$AB = \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -14 & -18 \\ -22 & -26 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 13 & -42 \end{bmatrix}$$

b) $$AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -13 & 11 \\ 25 & -19 & 20 \\ 13 & -11 & 16 \end{bmatrix}$$

c) $$AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix} = \begin{bmatrix} -20 \\ -27 \end{bmatrix}$$

$$BA = \underset{3\times 1}{\begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix}}\underset{2\times 3}{\begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}}$$ undefined

47. a) $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ if $ad-bc \neq 0$

$$A = \begin{bmatrix} 1 & -6 \\ 4 & -7 \end{bmatrix}$$

$$\det(A) = \frac{1}{1(-7)-4(-6)} \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$$

$$= \frac{1}{17} \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$$

b) $B = \begin{bmatrix} 12 & -3 \\ -8 & 2 \end{bmatrix}$

$$ad - bc = 12(2) - (-3)(-8) = 0$$

B^{-1} does not exist

48.

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 5 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 5 & 1 & -1 & 2 & 0 \end{array}\right] \xrightarrow{R_3 - 5R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -4 & 4 & 2 & -5 \end{array}\right]$$

$$\xrightarrow{R_1 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & -1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -4 & 4 & 2 & -5 \end{array}\right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & -1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1/2 & 5/4 \end{array}\right]$$

$$\xrightarrow[R_2 - R_3]{R_1 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 & -1/4 \\ 0 & 0 & 1 & -1 & -1/2 & 5/4 \end{array}\right]$$

so $A^{-1} = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1/2 & -1/4 \\ -1 & -1/2 & 5/4 \end{bmatrix}$

49. a)

$$\underset{A}{\begin{bmatrix} 8 & -1 \\ -4 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{C}{\begin{bmatrix} 17 \\ -5 \end{bmatrix}}$$

$$A^{-1} = \frac{1}{8-4} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 \\ 1 & 2 \end{bmatrix}$$

$$X = A^{-1} C = \begin{bmatrix} 1/4 & 1/4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 17 \\ -5 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$x = 3, \; y = 7$$

b)

$$\underset{A}{\begin{bmatrix} 4 & 0 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{C}{\begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - 4R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -4 & -4 & 1 & -4 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right] \xrightarrow[-R_3]{-\frac{1}{4}R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1/4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow[R_2 - R_3]{R_1 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & -1 & 1 \\ 0 & 1 & 0 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$$X = A^{-1} C = \begin{bmatrix} 1/4 & -1 & 1 \\ -1/4 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$x = 1, \; y = -1, \; z = -2$$

50. a)

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 2 & -1 & 4 & 6 \\ -3 & 2 & -3 & -1 \end{array}\right] \xrightarrow[R_3+3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 0 & -7 & 8 & -12 \\ 0 & 11 & -9 & 26 \end{array}\right]$$

$$\xrightarrow{-1/7\,R_2} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 11 & -9 & 26 \end{array}\right] \xrightarrow[R_3-11R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 10/7 & 27/7 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 0 & 25/7 & 50/7 \end{array}\right]$$

$$\xrightarrow{\frac{7}{25}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 10/7 & 27/7 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2+\frac{8}{7}R_3]{R_1-\frac{10}{7}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$x = 1, \quad y = 4, \quad z = 2$$

b)

$$\left[\begin{array}{ccc|c} 3 & -18 & 21 & 12 \\ 2 & 7 & -6 & 3 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 1 & -25 & 27 & 9 \\ 2 & 7 & -6 & 3 \end{array}\right]$$

$$\xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & -25 & 27 & 9 \\ 0 & 57 & -60 & -15 \end{array}\right] \xrightarrow{\frac{1}{57}R_2} \left[\begin{array}{ccc|c} 1 & -25 & 27 & 9 \\ 0 & 1 & -\frac{20}{19} & -\frac{5}{19} \end{array}\right]$$

$$\xrightarrow{R_1+25R_2} \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 13/19 & 46/19 \\ 0 & 1 & -20/19 & -5/19 \end{array}\right] \\ \begin{array}{cccc} x & y & z & \end{array} \end{array}$$

$$x + \frac{13}{19}z = \frac{46}{19}$$

$$y - \frac{20}{19}z = -\frac{5}{19}$$

z = anything

let $z = t$ then the general solution is

$$\begin{cases} x = \frac{46}{19} - \frac{13}{19}t \\ y = -\frac{5}{19} + \frac{20}{19}t \\ z = t \end{cases}$$

if $t = 0$: $x = \frac{46}{19}$, $y = -\frac{5}{19}$, $z = 0$

if $t = 1$: $x = \frac{33}{19}$, $y = \frac{15}{19}$, $z = 1$

c) $\left[\begin{array}{ccc|c} 1 & 3 & 3 & 12 \\ 2 & 20 & 10 & 8 \\ 1 & 10 & 5 & 0 \end{array}\right] \xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 3 & 5 & 12 \\ 0 & 0 & 0 & 8 \end{array}\right]$ ✗

no solution