Mass-spring systems

Tuesday, March 7, 2017

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$(m\ddot{x} + b\dot{x} + kx = 0)$$

auxiliary equation:

$$mn^2 + bn + k = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

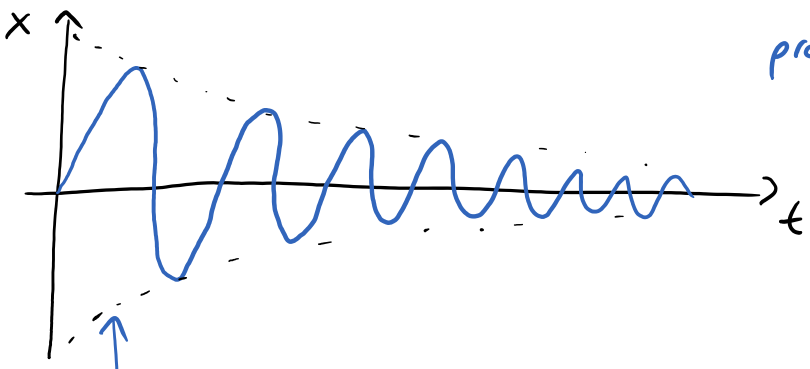
solutions will be

1. 2 distinct real if $b^2 - 4km > 0$
2. 1 repeated real " $= 0$
3. 2 complex " < 0

1. $x_1 = c_1 e^{n_1 t} + c_2 e^{n_2 t}$
2. $x_2 = (c_1 + c_2 t) e^{nt}$
3. $x_3 = e^{at} (c_1 \cos bt + c_2 \sin bt)$

what do these solutions look like?

3

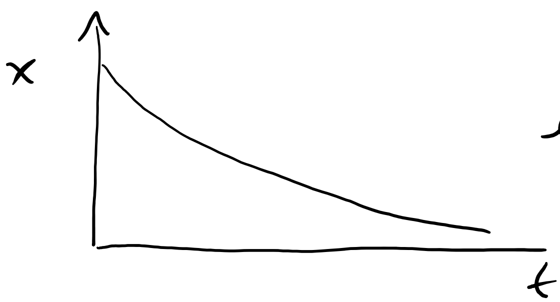


provided a is negative, so e^{at} is exponential decay

sinusoidal with decaying amplitude

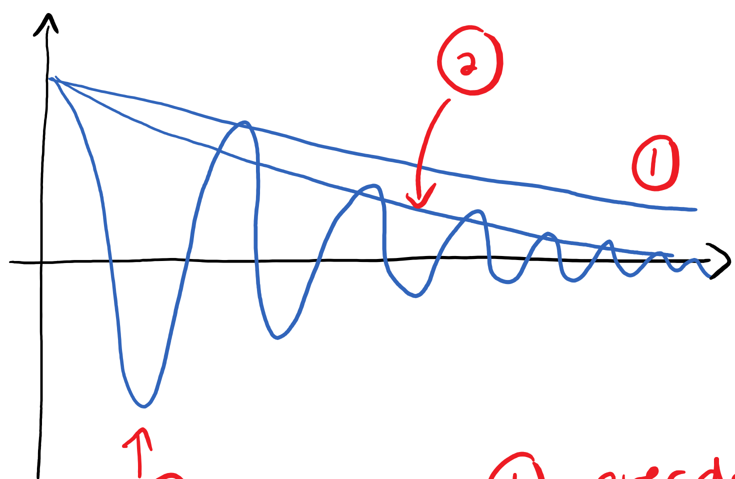
but what do 1 and 2 look like?

if the r's (solns to the aux eqn) are negative



exponential decay

so putting it all together:



1 overdamped
2 critically damped
3 underdamped

critically damped - just enough friction to prevent oscillation

→ object "returns to equilibrium" in minimum time

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so, what about an external force?

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = \underbrace{F_{ext}(t)}$$

Fexternal is a function of time

note: resonance happens when your external force is pumping with a frequency that approaches the natural frequency of the oscillator

→ amplitude of oscillation increases exponentially
