Math 193: Writing Differential Equations

Write a differential equation for each of the following scenarios.

1. The population of a town grows at a rate proportional to the population present P at time t.

$$\frac{dP}{dt} \propto P \qquad \text{so} \qquad \frac{dP}{dt} = kP$$

2. A radioactive isotope decays at a rate proportional to the amount present A at time t.

$$\frac{dA}{dt} \propto -A \qquad \text{so} \qquad \frac{dA}{dt} = -kA$$

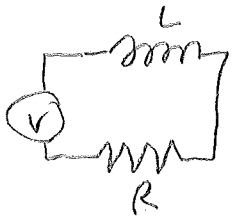
3. The rate at which the temperature of an object changes is proportional to the difference between the temperature of the body T and the temperature of the surrounding medium T_{room} (the room temperature).

$$\frac{dT}{dt} \propto (T - T_{room}) \qquad \text{so} \qquad \frac{dT}{dt} = k(T - T_{room})$$

4. If A is the amount of salt in a well-stirred tank, then the rate at which A changes is equal to the difference between the input rate of salt R_{in} and the output rate of salt R_{out}.

$$\frac{dA}{dt} = R_{in} - R_{out}$$

5. Consider a simple series circuit containing an inductor L and a resistor R. It is known that the voltage drop across each device is given by $L\frac{dI}{dt}$ and IR, respectively. If the circuit has voltage source V, write the equation for the voltage drop around the closed loop.



$$\sum V = 0$$

$$V = L\frac{dI}{dt} + IR$$

6. Consider a population P of fish. If this population is allowed to grow unchecked (unlimited food, no disease, no predation, etc.), the rate of growth for that population at time t is proportional to the population at that time.

   (a) Write down the differential equation for the rate of change of the fish population given the scenario above.

$$\frac{dP}{dt} = kP$$

   (b) Let's suppose that the fish are now also being harvested at a constant rate h, where h is a positive constant. Modify your DE from part (a) to include this new term.

$$\frac{dP}{dt} = kP - h$$

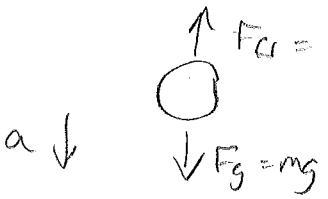
7. A contagious disease is spread throughout a community by people coming into contact with other people. Let x be the number of people who have the flu and y be the number of poeple who have not yet been exposed to the flu. Assume that the rate at which the disease spreads is proportional to the product of the number of people with the flu and the number without it.

$$\frac{dx}{dt} \propto xy \qquad \text{so} \quad \frac{dx}{dt} = kxy$$

$$= kx(n_{tot} - x)$$

for fixed population

8. An object in freefall has air resistance $F_{\text{fr}} = -kv$. If it is falling downwards under the influence of gravity, draw the free-body diagram and write the equation of motion.



Newton's Law:

$$\Sigma F = ma$$

$$F_g - F_{cr} = ma$$

$$mg - kv = m\frac{dv}{dt}$$