Dropping the Absolute Value in DE's

ex. Solve $\frac{dy}{dx} = 3y$ given that $y=-2$ when $x=0$.

$$dy = 3y\,dx$$
$$\int \frac{1}{y}\,dy = \int 3\,dx$$

Method 1: with absolute value

$$\int \frac{1}{y}\,dy = \int 3\,dx$$
$$\ln|y| = 3x + C \quad \leftarrow \text{general solution in implicit form}$$

Option 1: find C first, then solve for y

$$\ln|y| = 3x + C$$
$$\ln|-2| = 3(0) + C$$
$$\ln 2 = 0 + C$$
$$C = \ln 2$$

particular solution in implicit form $\longrightarrow$ so $\ln|y| = 3x + \ln 2$

now solve for y:

$$|y| = e^{3x+\ln 2}$$
$$|y| = e^{3x}e^{\ln 2}$$
$$|y| = e^{3x}\,2$$
$$y = \pm 2e^{3x}$$

if $y = 2e^{3x}$

$-2 = 2e^{3(0)}$

$-2 = 2$ ✗

if $y = -2e^{3x}$

$-2 = -2e^{3(0)}$

$-2 = -2$ ✓

particular sol. in explicit form $\longrightarrow$ so $\boxed{y = -2e^{3x}}$

Option 2: solve for y first, then find C

$$\ln|y| = 3x + C$$
$$|y| = e^{3x+C}$$
$$|y| = e^{3x}e^{C}$$
$$y = \pm e^{C}e^{3x} \quad \leftarrow \text{general solution in explicit form}$$

if $y = e^{C}e^{3x}$

$-2 = e^{C}e^{3(0)}$

$-2 = e^{C}$

$\ln(-2) = C$

no solution

if $y = -e^{C}e^{3x}$

$-2 = -e^{C}e^{3(0)}$

$-2 = -e^{C}$

$2 = e^{C}$

$\ln 2 = C$

so $y = -e^{\ln 2}e^{3x}$

$\boxed{y = -2e^{3x}}$ $\leftarrow$ particular solution in explicit form

Method 2: without absolute value

$$\int \frac{1}{y}\,dy = \int 3\,dx$$

$$\ln y = 3x + C$$

at this step, we cannot solve for C since $\ln(-2)$ is undefined. So we must solve for y first.

$$y = e^{3x+C}$$

$e^C > 0$ for all C

$$y = e^{3x}\boxed{e^{C}}$$

since e^C is a constant, we can simplify as $k = e^C$

but

$$y = ke^{3x}$$

general solution in explicit form

k is treated like any real (this replaces the role of the $\pm$ from the absolute value)

$$-2 = ke^{3(0)}$$

$$-2 = k$$

so $\boxed{y = -2e^{3x}}$

particular solution in explicit form