

Math 193: Useful Properties of Logs for DE

Two useful properties of logarithms:

$$\ln x^n = n \ln x$$

$$e^{\ln x} = x$$

Note that to use the property $e^{\ln x} = x$, the expression must be exactly in that form, with no coefficient in front of the logarithm. If there is a coefficient, you need to use the power rule first, as in the following examples.

$$e^{2 \ln x} = e^{\ln x^2} = x^2 \quad (\text{not } 2x)$$

$$e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x} \quad (\text{not } -x)$$

Exercises:

Rewrite the following using the power rule: $\ln x^n = n \ln x$.

1. $7 \ln 5$
2. $\ln 2^n$
3. $\ln \sqrt{m}$
4. $\ln \frac{1}{p}$
5. $-2 \ln k$

Simplify.

6. $e^{\ln m}$
7. $e^{\ln xy}$
8. $e^{2 \ln t}$
9. $e^{\frac{1}{2} \ln r}$
10. $e^{-4 \ln b}$
11. $e^{\ln \cos 2\theta}$
12. $e^{-\ln 2}$

Answers:

1. $7 \ln 5 = \ln 5^7$
2. $\ln 2^n = n \ln 2$
3. $\ln \sqrt{m} = \ln m^{1/2} = \frac{1}{2} \ln m$
4. $\ln \frac{1}{p} = \ln p^{-1} = -\ln p$
5. $-2 \ln k = \ln k^{-2}$ or $\ln \frac{1}{k^2}$
6. $e^{\ln m} = m$
7. $e^{\ln xy} = xy$
8. $e^{2 \ln t} = e^{\ln t^2} = t^2$
9. $e^{\frac{1}{2} \ln r} = e^{\ln r^{1/2}} = r^{1/2}$ or \sqrt{r}
10. $e^{-4 \ln b} = e^{\ln b^{-4}} = b^{-4}$ or $\frac{1}{b^4}$
11. $e^{\ln \cos 2\theta} = \cos 2\theta$
12. $e^{-\ln 2} = e^{\ln 2^{-1}} = 2^{-1}$ or $\frac{1}{2}$