Section 28.1: cont'd

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recall:

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \iff \frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

so what about

$$\frac{d}{dx}\left(\sin^{-1} 3x\right) = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{3}{\sqrt{1-9x^2}}$$

$$\frac{d}{dx}\left(\sin^{-1} \frac{x}{a}\right) = \frac{1}{\sqrt{1-(x/a)^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{1-x^2/a^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{(1-x^2/a^2)\,a^2}}$$

$$= \frac{1}{\sqrt{a^2-x^2}}$$

Similarly, $\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{1+(x/a)^2} \cdot \frac{1}{a}$

$$= \frac{1}{1 + x^2/a^2} \cdot \frac{1}{a}$$

$$= \frac{1}{a + x^2/a} \cdot \frac{a}{a}$$

$$= \frac{a}{a^2 + x^2}$$

So $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$