Section 28.2: The Basic Logarithm Form

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recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

where the domain of $\ln x$ is $x > 0$

values of x for which the function ln x is defined

so $\int \frac{1}{x}\,dx = \underbrace{\ln|x|}_{\text{this}} + C$

x can have any non-zero value

this ensures that the logarithm is defined

examples:

① $\int \frac{dx}{x-3} = \ln|x-3| + C$

note: can use substitution

let $u = x-3$

$du = dx$

integral $= \int \frac{du}{u} = \ln|u| + C$

$= \ln|x-3| + C$

② $\int \frac{x^3\,dx}{1-x^4}$

let $u = 1-x^4$

$du = -4x^3\,dx$

$\frac{du}{-4} = x^3\,dx$

$= \int \frac{du}{-4u}$

$$= \int \frac{du}{-4u}$$

$$= -\frac{1}{4} \ln |u| + C$$

$$= -\frac{1}{4} \ln |1 - x^4| + C$$

③ $$\int \frac{e^{-3x}}{2 + 5e^{-3x}}\, dx$$

let $u = 2 + 5e^{-3x}$
$du = -15e^{-3x}\,dx$
$\frac{du}{-15} = e^{-3x}\,dx$

$$= \int \frac{du}{-15u}$$

$$= -\frac{1}{15} \ln |u| + C$$

$$= -\frac{1}{15} \ln |2 + 5e^{-3x}| + C$$ ← perfectly acceptable answer #1

$$= -\frac{1}{15} \ln (2 + 5e^{-3x}) + C$$ ← perfectly acceptable answer #2

and, unfortunately, you can rewrite this expression:

$$= -\frac{1}{15} \ln \left(\frac{2e^{3x} + 5}{e^{3x}} \right) + C$$

note: $2 + 5e^{-3x}\left(\frac{e^{3x}}{e^{3x}}\right) = \frac{2e^{3x} + 5\cancel{e^{-3x}}\cancel{e^{3x}}}{e^{3x}}$

$$= -\frac{1}{15} \left[\ln (2e^{3x} + 5) - \ln e^{3x} \right] + C$$

$$= -\frac{1}{15} \left[\ln (2e^{3x} + 5) - 3x \right] + C$$

$$\ldots \ln (2e^{3x} + 5) + C$$ ← perfectly

$\frac{1}{15}\left[\quad \right]$

$= \frac{1}{5}x - \frac{1}{15}\ln(2e^{3x}+5) + C$ ← perfectly acceptable answer #3

④ $\displaystyle\int \frac{\sin 2\theta}{1-\cos^2\theta}\,d\theta$

method #1:

$= \displaystyle\int \frac{2\sin\theta\cos\theta\,d\theta}{1-\cos^2\theta}$

let $u = 1-\cos^2\theta$
$du = +2\cos\theta\sin\theta\,d\theta$

$= \displaystyle\int \frac{du}{u}$

$= \ln|u| + C$

$= \ln\left|1-\cos^2\theta\right| + C$

method #2:

$= \displaystyle\int \frac{2\sin\theta\cos\theta\,d\theta}{\sin^2\theta}$

$= \displaystyle\int \frac{2\cos\theta\,d\theta}{\sin\theta}$

let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$= \displaystyle\int \frac{2\,du}{u}$

$= 2\ln|u| + C$

$= 2\ln|\sin\theta| + C$