Section 28.6: Inverse Trig Forms

Monday, January 15, 2018 10:55 AM

Consider the integral $\int \frac{dx}{\sqrt{1-x^2}}$

regular substitution fails! (need x in numerator)

you probably recognize this already:

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

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digression: why? (will <u>not</u> be tested)

we need a different type of substitution:

$$\int \frac{dx}{\sqrt{1-x^2}}$$

let $\sin u = x$

$\cos u \, du = dx$

$\uparrow$ if $\sin u = x$

$u = \sin^{-1} x$

$$= \int \frac{\cos u \, du}{\sqrt{1-\sin^2 u}}$$

$$= \int \frac{\cos u \, du}{\sqrt{\cos^2 u}}$$

$$= \int \frac{\cos u \, du}{\cos u}$$

$$= \int du$$

$$= \int du$$

$$= u + C$$

$$= \sin^{-1} x + C$$

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in general

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

on formula sheet

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examples: warmups:

$$\int \frac{dx}{\sqrt{16 - x^2}} = \sin^{-1}\left(\frac{x}{4}\right) + C$$

$a = 4$

$$\int \frac{dx}{x^2 + 5} = \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$$

$a = \sqrt{5}$

$= $ if you insist, $\frac{\sqrt{5}}{5} \tan^{-1}\left(\frac{\sqrt{5}\, x}{5}\right) + C$

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examples:

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1. $$\int \frac{dx}{x^2+8x+25}$$

$$= \int \frac{dx}{x^2+8x+\underline{16}+25-\underline{16}}$$

$$= \int \frac{dx}{(x+4)^2+9}$$

so $a^2=9$ and $a=3$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x+4}{3}\right)+C$$