Section 28.6: cont'd

Tuesday, January 16, 2018 10:27 AM

② $\displaystyle\int \frac{dx}{\sqrt{6x - x^2}}$

$$= \int \frac{dx}{\sqrt{\underline{9} - (x^2 - 6x + \underline{9})}}$$

$$= \int \frac{dx}{\sqrt{9 - (x-3)^2}}$$

$a = 3$

$$= \sin^{-1}\left(\frac{x-3}{3}\right) + C$$

more examples:

$$\int \frac{5p^2}{9 + p^6}\, dp$$

let $u = p^3$

$du = 3p^2\,dp$

$\frac{du}{3} = p^2\,dp$

$$= \int \frac{5}{3} \frac{du}{9 + u^2}$$

$a = 3$

$$= \frac{5}{3} \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{5}{9} \tan^{-1}\left(\frac{p^3}{3}\right) + C$$

compare and contrast:

$$\int \frac{x\,dx}{x^2+1} \quad \text{vs} \quad \int \frac{dx}{x^2+1} \quad \text{vs} \quad \int \frac{x+1}{x^2+1}\,dx$$

$\int \frac{x\,dx}{x^2+1}$: substitution, let $u = x^2+1$

$\int \frac{dx}{x^2+1}$: arctan

$\int \frac{x+1}{x^2+1}\,dx$: break into

$$\int \left(\frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$\int \frac{e^{3x}}{4+e^{6x}}\,dx \quad \text{vs} \quad \int \frac{e^{3x}}{4+e^{3x}}\,dx$$

let $u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{du}{3} = e^{3x}\,dx$

$$= \int \frac{1}{3} \frac{du}{4+u^2}$$

$a = 2$

$$= \frac{1}{3} \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) + C$$

$\frac{1}{a}$

$$= \frac{1}{6} \tan^{-1}\left(\frac{e^{3x}}{2}\right) + C$$

let $u = 4 + e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{du}{3} = e^{3x}\,dx$

$$= \int \frac{1}{3} \frac{du}{u}$$

$$= \frac{1}{3} \ln|u| + C$$

$$= \frac{1}{3} \ln\left(e^{3x}+4\right) + C$$