Section 29.3: Partial Derivatives

Tuesday, January 23, 2018 11:02 AM

what happens we try to take the derivative of a function of two or more variables?

→ these derivative's are called "partial derivatives"

and are written:

$$\frac{\partial f}{\partial x}$$

∂ is like a backward six, not a "d" (dee)

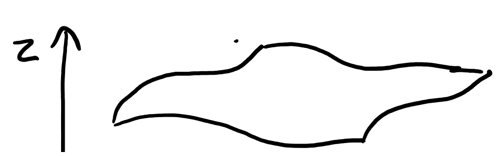
other notations:

$$f_x(x,y) \quad , \quad f_y(x,y)$$

$$\frac{\partial}{\partial x} f(x,y) \quad , \quad \frac{\partial}{\partial y} f(x,y)$$

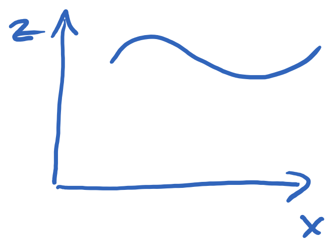
note: must specify which variable you are differentiating with respect to!

main idea:



← some curved surface

if we take a slice parallel to the x-z plane



the slope of this line is $\frac{\partial z}{\partial x}$

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so, how do you calculate partial derivatives?

- treat the other variable(s) as if they were constants and differentiate as usual

example: find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = x^2 + 2xy$

$$\frac{\partial f}{\partial x} = 2x + 2y$$

$$\frac{\partial f}{\partial y} = 0 + 2x = 2x$$

note: the actual full definition of partial derivatives:

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

but we won't get into that

example: find $\left. \frac{\partial z}{\partial y} \right|_{(3, \pi/2, 9)}$ for $z = x^2 \cos 4y$

$$\frac{\partial z}{\partial y} = -4x^2 \sin 4y$$

$$\left. \frac{\partial z}{\partial y} \right|_{(3, \pi/2, 9)} = -4(3)^2 \sin\left(4 \cdot \pi/2\right) = 0$$