Section 31.1: Solutions of Differential Equations

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differential equation (DE) $\equiv$ an equation that contains derivatives or differentials

example:

$$\frac{dy}{dx} = x^2 + 3 \qquad\qquad y'' + 3y' - 2y = x^2$$

jargon:

if the equation contains only first derivatives, it's called a first order DE

if the equation contains second derivatives, it's a second order DE

$\therefore$ the order of the equation = order of the highest derivative in the equation

(if you look at the book, it also talks about the degree, but most DE texts just look at whether the equation is linear or not
→ is the derivative raised to a power)

example:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 7 \quad \leftarrow 2^{nd} \text{ order}$$

$$y''' = y' + 2 \quad \leftarrow 3^{rd} \text{ order}$$

solution to a DE $\equiv$ a relation between variables that satisfies the DE

note: doesn't have to be a function

example: $\underbrace{\frac{dy}{dx} = x^2 + 3}_{\text{DE}}$ has solution $\underbrace{y = \frac{x^3}{3} + 3x + C}_{\text{solution has no derivatives or differentials}}$

general solution $\equiv$ a solution to a DE that contains a number of arbitrary constants equal to the order of that DE

so for 2^{nd} order DE $\rightarrow$ 2 arbitrary constants

particular solution - when specific values are given to at least one of the constants

example: $\frac{dy}{dx} = x^2 + 3$ and $y = \frac{x^3}{3} + 3x + 2$

example: show that $y = c \ln x$ (c = a constant) is a solution to the DE

$$y' \ln x - \frac{y}{x} = 0$$

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how do you do this? differentiate the solution and then substitute into the DE to get an identity (equation that's true for all x)

differentiate $\quad y = c \ln x$

$$y' = \frac{c}{x}$$

now substitute into

$$y' \ln x - \frac{y}{x} = 0$$

$$\frac{c}{x} \ln x - \frac{c \ln x}{x} = 0 \quad \checkmark$$

note: this allows you to check your work!