Section 31.4 cont'd

Monday, February 5, 2018 10:25 AM

linear DE of first order

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

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example: Are the following DE's linear first order? If so, give $P(x)$ and $Q(x)$.

d) $yy' + y = \sin x$ No!

$\Rightarrow y' + \frac{y}{y} = \frac{\sin x}{y}$ y is in denom

e) $\frac{d\theta}{dt} + \theta \sin t = e^t$ Yes!

with $P(t) = \sin t$
$Q(t) = e^t$

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skillbuilder:

$$\frac{d}{dx}(ye^x) = \frac{dy}{dx} e^x + y e^x$$

$$\frac{d}{dx}(yx^2) = \frac{dy}{dx} x^2 + 2yx = \frac{dy}{dx} x^2 + 2xy$$

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how do you solve this type of DE? (linear, first order)

- follow the steps in the handout

full example: solve

$$dy - 3y\,dx = e^{3x}\,dx$$

$$\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$$

$$\frac{dy}{dx} - 3y = e^{3x}$$

this is linear with $P(x) = -3$
$Q(x) = e^{3x}$

now, find integrating factor

$$IF = e^{\int P(x)\,dx} = e^{\int -3\,dx}$$

$$= e^{-3x}$$

note: omit $+C$

then multiply both sides of DE by the integrating factor

$$\frac{dy}{dx} - 3y = e^{3x}$$

$-3x \quad - \quad -3x \quad 3x \quad -3x$

$$\frac{dy}{dx} e^{-3x} - 3y e^{-3x} = e^{3x} e^{-3x}$$

$$\frac{d}{dx}\left(y e^{-3x}\right) = 1$$

so, then

$$d\left(y e^{-3x}\right) = dx \qquad \text{(multiply by } dx\text{)}$$

now, integrate

$$\int d\left(y e^{-3x}\right) = \int dx$$

$$y e^{-3x} = x + C$$

if you need to give an explicit solution, then

$$y = e^{3x}(x + C)$$