Review:

Friday, February 9, 2018 10:49 AM

Solve the following 1st order linear DE:

$$\cos x \frac{dy}{dx} = 1 - y \sin x$$

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

$$\frac{dy}{dx} + y \tan x = \sec x$$

$$IF = e^{\int \tan x \, dx} = e^{\ln(\sec x)} = \sec x$$

$$= e^{-\ln \cos x} = e^{\ln \frac{1}{\cos x}} = \frac{1}{\cos x}$$

$$\sec x \frac{dy}{dx} + y \tan x \sec x = \sec^2 x$$

$$\frac{d}{dx}(y \sec x) = \sec^2 x$$

$$\int d(y \sec x) = \int \sec^2 x \, dx$$

$$\boxed{y \sec x = \tan x + C}$$

give an explicit solution:

$$y = \cos x \tan x + C \cos x$$

$= \sin x + C \cos x$

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solve the following linear first order DE

$$dy = (1 - 2y)\, x\, dx$$

using a) linear first order
b) separation of variables

a) $\frac{dy}{dx} = x - 2xy$

$\frac{dy}{dx} + 2xy = x$

$IF = e^{\int P(x)dx} = e^{\int 2x\,dx}$

$= e^{x^2}$

$\frac{dy}{dx} e^{x^2} + 2xy\, e^{x^2} = x\, e^{x^2}$

$\frac{d}{dx}\left(y e^{x^2}\right) = x e^{x^2}$

$\int d\left(y e^{x^2}\right) = \int x e^{x^2} dx$

$$\boxed{y e^{x^2} = \frac{1}{2} e^{x^2} + C}$$

if you like, $y = \frac{1}{2} + ce^{-x}$

b) $dy = (1-2y)\, x\, dx$

$$\frac{dy}{1-2y} = x\, dx$$

$$\boxed{-\frac{1}{2}\ln(1-2y) = \frac{1}{2}x^2 + C}$$

if you like

$$\ln(1-2y) = -x^2 - 2c$$

$$1-2y = e^{-x^2-2c}$$

$$1 - e^{-x^2-2c} = 2y$$

$$\frac{1}{2} - \frac{1}{2}e^{-x^2-2c} = y$$

$$\frac{1}{2} - e^{-x^2}\frac{e^{-2c}}{2} = y$$

$$y = \frac{1}{2} - c_1 e^{-x^2}$$