Section 31.9: Solutions of Nonhomogeneous 2nd order Linear DEs

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nonhomogeneous:

$$ay'' + by' + cy = \underbrace{f(x)}_{\text{this was zero for homogeneous DE's}}$$

here's the plan:

we find y_c ← called the complementary solution - it's the general solution to the associated homogeneous DE

how? Sections 31.7 & 31.8: set RHS to zero and solve - this solution is y_c

then

$$y = y_c + y_p$$

y: the solution to the DE

y_c: solution to the homogeneous DE

y_p: particular solution necessary to get the RHS of the non-homogeneous

DE

the non-homogeneous DE

note: y_p will have <u>no</u> arbitrary constants

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example: solve $y'' + 9y = 8e^x$

complementary:

$$y'' + 9y = 0$$

aux eqn: $m^2 + 9 = 0$

$$m = \pm 3i$$

$$= a + bi \quad \text{where } a = 0, \; b = 3$$

$$y_c = e^{ax}(C_1 \cos bx + C_2 \sin bx)$$

$$= C_1 \cos 3x + C_2 \sin 3x$$

particular: RHS $= 8e^x$

$$y_p = Ae^x$$

now differentiate and then plug back into DE:

$$y_p' = Ae^x$$

$$y_p'' = Ae^x$$

DE:

$$y'' + 9y = 8e^x$$

$$Ae^x + 9Ae^x = 8e^x$$

$$10Ae^x = 8e^x$$

$$10A = 8$$

$$A = 4/5$$

$$y_p = \frac{4}{5}e^x$$

complete solution:

$$y = y_c + y_p$$

$$\boxed{y = c_1 \cos 3x + c_2 \sin 3x + \frac{4}{5}e^x}$$