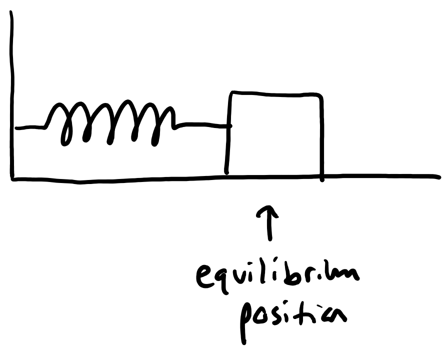
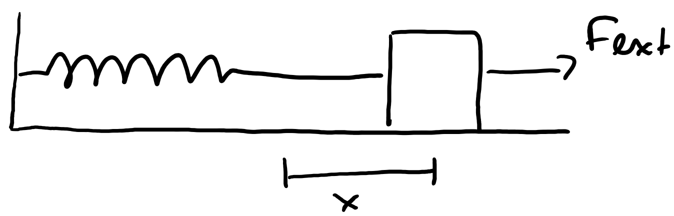
Section 31.10: Applications of Higher-Order Equations

Wednesday, February 28, 2018 10:57 AM

Hooke's Law:



consider a mass on a frictionless surface with an ideal spring



pull spring to the right by Fext (a constant external force)

- spring pulls back, stretches until

$$\vec{F}_{spring} = -\vec{F}_{ext}$$

but Hooke's Law says

$$\vec{F}_{spring} = -k\vec{x}$$

if you stretch the spring to right, force is to the left (opposite direction to the displacement) now, let go of the block so $F_{ext} = 0$

what happens?

block has <u>unbalanced</u> force $\vec{F}_{spring}$ pulling back towards equilibrium position

$$\sum \vec{F} = m\vec{a}$$

$$-kx = m\frac{d^2x}{dt^2}$$ ← 2^{nd} order linear DE

rearranging:

$$m\frac{d^2x}{dt^2} + kx = 0$$ homogeneous

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$ $(y'' + 2y' - 3y = 0)$

aux eqn: but don't use m's - use n

$$n^2 + \frac{k}{m} = 0$$

$$n^2 = -\frac{k}{m}$$

$$n = \pm\sqrt{\frac{k}{m}}\,i$$

so $n = a \pm bi$ where $a = 0$ and $b = \sqrt{\frac{k}{m}}$

$$x = \cancel{e^{at}}^{1}(c_1 \cos bt + c_2 \sin bt)$$

$$x = \cancel{e^{at}} (c_1 \cos bt + c_2 \sin bt)$$

$$= c_1 \cos \sqrt{\frac{k}{m}}\, t + c_2 \sin \sqrt{\frac{k}{m}}\, t$$

you may have seen this before as

$$x = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

where ω_0 = natural frequency of oscillation

$$\omega_0 = \sqrt{\frac{k}{m}}$$