Section 31.9: Cont'd

Wednesday, February 28, 2018 10:31 AM

initial conditions:

solve

$$y'' - 2y' + y = xe^{2x} - e^{2x}$$

given that $y' = 4$ and $y = -2$ when $x = 0$.

complementary solution:

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1 \qquad \text{one repeated sol'n}$$

$$y_c = (c_1 + c_2 x)e^x$$

particular solution: RHS: $xe^{2x} - e^{2x}$

$$y_p = Axe^{2x} + Be^{2x} \qquad \text{NOT BAD CASE}$$

$$y_p' = 2Axe^{2x} + Ae^{2x} + 2Be^{2x}$$

$$y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} + 4Be^{2x}$$

$$= 4Axe^{2x} + 4Ae^{2x} + 4Be^{2x}$$

plug back into DE:

$$y'' - 2y' + y = xe^{2x} - e^{2x}$$

$$(4Axe^{2x} + 4Ae^{2x} + 4Be^{2x}) - 2(2Axe^{2x} + Ae^{2x} + 2Be^{2x})$$

$$+ Axe^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$\cancel{4Axe^{2x}} + 4Ae^{2x} + \cancel{4Be^{2x}} - \cancel{4Axe^{2x}} - 2Ae^{2x} - \cancel{4Be^{2x}}$$
$$+ Axe^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$Axe^{2x} + 2Ae^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$Axe^{2x} + (2A + B)e^{2x} = xe^{2x} - e^{2x}$$

so $A = 1$ and $2A + B = -1$

$$B = -1 - 2A$$
$$= -3$$

$$\boxed{y_p = xe^{2x} - 3e^{2x}}$$

full solution: $y = y_c + y_p$

$$= (c_1 + c_2 x)e^{x} + xe^{2x} - 3e^{2x}$$

so initial conditions: when $x = 0$, $y = -2$

$$y = (c_1 + c_2 x)e^{x} + xe^{2x} - 3e^{2x}$$
$$-2 = (c_1 + 0)\cdot 1 + 0 - 3$$
$$c_1 = 1$$

and when $x = 0$, $y' = 4$

$$y' = (c_1 + c_2 x)e^{x} + c_2 e^{x} + 2xe^{2x} + e^{2x} - 6e^{2x}$$
$$4 = (c_1 + 0)\cdot 1 + c_2 + 0 + 1 - 6$$

$$c_2 = 8$$

$$\boxed{y = (1+8x)e^{x} + (x-3)\, e^{2x}}$$ particular solution