Section 31.10: cont'd

Thursday, March 1, 2018 10:38 AM

See handout of questions:

1) $$\frac{d^2s}{dt^2} + 4s = 0$$

a) auxiliary eqn: $m^2 + 4 = 0$

$m = \pm 2i = a \pm bi$

$$s = c_1 \cos 2t + c_2 \sin 2t$$

b) at $t=0$, $s = 2.5$ cm and $\frac{ds}{dt} = 0$

so $s = c_1 \cos 2t + c_2 \sin 2t$

$2.5 = c_1 \cancel{\cos 0}^{1} + c_2 \cancel{\sin 0}^{0}$

$c_1 = 2.5$

and $\frac{ds}{dt} = -2c_1 \sin 2t + 2c_2 \cos 2t$

$0 = -2c_1 \cancel{\sin 0}^{0} + 2c_2 \cancel{\cos 0}^{1}$

$c_2 = 0$

so $\boxed{s = 2.5 \cos 2t}$

c) new initial conditions

when $t=0$, $s=0$

$t=0$, ds 6 cm/s

dt =

$$s = c_1 \cos 2t + c_2 \sin 2t$$

$$0 = c_1 \cancelto{1}{\cos 0} + c_2 \cancelto{0}{\sin 0}$$

$$c_1 = 0$$

$$s = c_2 \sin 2t$$

$$\frac{ds}{dt} = 2 c_2 \cos 2t$$

$$6 = 2 c_2 \cancelto{1}{\cos 0}$$

$$c_2 = 3$$

$$\boxed{s = 3 \sin 2t}$$

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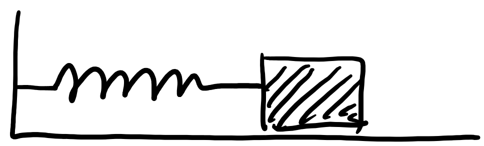
back to Hooke's Law:

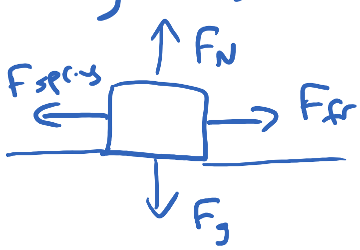


what if there is air resistance?

$$F_{fr} \propto v$$

$$F_{fr} = b v$$

free-body diagram



where direction of F_{fr} depends on direction of velocity.

$\vec{F}_{fr} = -b\vec{v}$

where direction of F_{spring} is opposite to $\vec{x}$

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{spring} + \vec{F}_{drag} = m\vec{a}$$

$$-kx - bv = ma$$

$$0 = ma + bv + kx$$

$$\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$$

$(m\ddot{x} + b\dot{x} + kx = 0)$

(dots mean "differentiate with respect to t, while primes mean "differentiate with respect to x)

So on formula sheet, we give

$$ma = -\beta v - kx + \underbrace{f(t)}_{\text{external force}}$$