Section 28.2: The Basic Logaritm Form

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recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

where the $\underbrace{\text{domain}}_{\substack{\text{values of } x \text{ for which} \\ \ln x \text{ is defined}}}$ of $\ln x$ is $x > 0$

so $\int \frac{1}{x}\,dx = \ln|x| + C$

x can have any value

examples:

① $\int \frac{dx}{x-3} = \ln|x-3| + C$

② $\int \tan\theta \, d\theta$

$= \int \frac{\sin\theta}{\cos\theta}\,d\theta$

$= \int \frac{-du}{u}$

$= -\ln|u| + C$

let $u = \cos\theta$
$du = -\sin\theta\, d\theta$
$-du = \sin\theta\, d\theta$

$= -\ln|\cos\theta| + C$ $\leftarrow$ perfectly acceptable answer #1

$= \ln|\sec\theta| + C$ $\leftarrow$ perfectly acceptable answer #2

note: similarly,

$\int \cot\theta \, d\theta = \ln|\sin\theta| + C$ (textbook)

$= -\ln|\csc\theta| + C$ (elsewhere)

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(3) $\int \frac{x^3 \, dx}{1-x^4}$

let $u = 1 - x^4$
$du = -4x^3 \, dx$

$= \int \frac{du}{-4u}$

$= -\frac{1}{4} \ln|u| + C$

$= -\frac{1}{4} \ln|1-x^4| + C$

(4) $\int \frac{e^{-3x}}{2+5e^{-3x}} \, dx$

let $u = 2+5e^{-3x}$
$du = -15e^{-3x} \, dx$

$= \int -\frac{1}{15} \frac{du}{u}$

$$= -\frac{1}{15} \ln |u| + C$$

$$= -\frac{1}{15} \ln |2 + 5e^{-3x}| + C$$ perfectly acceptable answer #1

$$= -\frac{1}{15} \ln \underbrace{(2 + 5e^{-3x})}_{\text{always positive}} + C$$ answer #2

and, unfortunately, you can rewrite this to get:

$$= -\frac{1}{15} \ln \left(\frac{2e^{3x} + 5}{e^{3x}} \right) + C$$

$$= -\frac{1}{15} \left[\ln (2e^{3x} + 5) - \ln e^{3x} \right] + C$$

$$= -\frac{1}{15} \left[\ln (2e^{3x} + 5) - 3x \right] + C$$

$$= \frac{1}{5}x - \frac{1}{15} \ln (2e^{3x} + 5) + C$$ answer #3

5 $$\int \frac{\sin 2\theta}{1 - \cos^2\theta} \, d\theta$$

$$= \int \frac{2 \sin\theta \cos\theta}{1 - \cos^2\theta} \, d\theta$$

method #1:

let $u = 1-\cos^2\theta$
$du = -2\cos\theta(-\sin\theta)\,d\theta$
$= 2\sin\theta\cos\theta\,d\theta$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|1-\cos^2\theta| + C$$

method #2:

$$= \int \frac{2\cancel{\sin}\theta\cos\theta\,d\theta}{\sin^{\cancel{2}}\theta}$$

$$= \int 2\frac{\cos\theta}{\sin\theta}\,d\theta$$

let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$$= \int \frac{2\,du}{u}$$

$$= 2\ln|u| + C$$

$$= 2\ln|\sin\theta| + C$$

note: $\ln|1-\cos^2\theta|$

$$= \ln|\sin^2\theta|$$

$$= 2\ln|\sin\theta|$$