Section 28.3: The Exponential Form

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recall: $\frac{d}{dx}(e^x) = e^x$

therefore: $\int e^x\,dx = e^x + C$

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examples:

① $\int e^{3x}\,dx$

let $u = 3x$
$du = 3\,dx$

$= \int \frac{1}{3} e^u\,du$

$= \frac{1}{3} e^u + C$

$= \frac{1}{3} e^{3x} + C$

in general: $\boxed{\int e^{ax}\,dx = \frac{e^{ax}}{a} + C \quad \text{for } a \neq 0}$

② $\int x e^{x^2}\,dx$

let $u = x^2$
$du = 2x\,dx$

$= \int \frac{e^u\,du}{2}$

$$= \frac{e^u}{2} + c$$

$$= \frac{e^{x^2}}{2} + C$$

note: $\displaystyle\int_0^1 x e^{x^2}\, dx = \frac{1}{2} e^{x^2} \Big|_0^1 = \frac{1}{2}(e-1)$

(3) $\displaystyle\int \cos 2\theta \; e^{\sin\theta \cos\theta} \, d\theta$

$$= \int \cos 2\theta \; e^{\frac{1}{2}\sin 2\theta} \, d\theta$$

$$= \int e^u \, du$$

let $u = \frac{1}{2}\sin 2\theta$
$du = \frac{2\cos 2\theta}{2}\, d\theta$

$$= e^u + c$$

$$= e^{\frac{1}{2}\sin 2\theta} + C$$

(4) $\displaystyle\int \sqrt{e^{3y} - 3e^{2y}} \, dy$

(tricky)

$\sqrt{e^{2y}} = \left(e^{2y}\right)^{1/2}$
$= e^y$

$$= \int \sqrt{e^{2y}(e^y - 3)} \, dy$$

$$= \int e^y \sqrt{e^y - 3} \, dy$$

let $u = e^y - 3$

let $u = e^y - 3$

$du = e^y\,dy$

$$= \int u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(e^y - 3)^{3/2} + C$$

⑤ $$\int (e^x + e^{-x})^2\,dx$$

similarly annoying

$$= \int (e^x e^x + 2e^x e^{-x} + e^{-x}e^{-x})\,dx$$

$$= \int (e^{2x} + 2 + e^{-2x})\,dx$$

$$= \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$$

⑥ $$\int \frac{4\,d\theta}{\sec\theta\; e^{\sin\theta}}$$

differently annoying

$$= \int 4\cos\theta\, e^{-\sin\theta}\,d\theta$$

let $u = -\sin\theta$

$du = -\cos\theta\,d\theta$

let $u = -\sin\theta$
$du = -\cos\theta \, d\theta$

$$= \int -4 e^{u} \, du$$

$$= -4 e^{u} + C$$

$$= -4 e^{-\sin\theta} + C$$

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(7) $\int \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx$

let $u = -\sqrt{x}$

$du = -\frac{1}{2} x^{-1/2} \, dx$

$du = -\frac{1}{2\sqrt{x}} \, dx$

$-2du = \frac{1}{\sqrt{x}} \, dx$

$$= \int -2 e^{u} \, du$$

$$= -2 e^{u} + C$$

$$= -2 e^{-\sqrt{x}} + C$$