Section 28.6: Inverse Trig Forms

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note: omit $\boxed{28.5}$

consider the integral $\int \frac{dx}{\sqrt{1-x^2}}$

regular substitution fails!
→ need "x" in numerator

you probably recognize this already

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

<u>digression</u> (will not be tested)

$$\int \frac{dx}{\sqrt{1-x^2}}$$

Let $\left.\begin{aligned} \sin u &= x \\ \cos u \, du &= dx \end{aligned}\right\}$ Section 28.8

if $\sin u = x$
then $u = \sin^{-1} x$

$$= \int \frac{\cos u \, du}{\sqrt{1-\sin^2 u}}$$

$$= \int \frac{\cos u \, du}{\sqrt{\cos^2 u}}$$

$$= \int \frac{\cancel{\cos u} \, du}{\cancel{\cos u}}$$

$$= \int du$$

$$= u + C$$

$$= \sin^{-1} x + C$$

end digression

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in general:

$$\left.\begin{aligned} \int \frac{dx}{\sqrt{a^2 - x^2}} &= \sin^{-1}\left(\frac{x}{a}\right) + C \\ \int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \end{aligned}\right\} \text{formula sheet}$$

examples:

warmups:

$$\int \frac{dx}{\sqrt{16-x^2}} = \sin^{-1}\left(\frac{x}{4}\right) + C$$

$$\int \frac{dx}{x^2+5} = \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$$

$$\int \frac{dx}{x^2+8x+25}$$

take coeff on x, divide by two, and square it

$$= \int \frac{dx}{x^2+8x+16+25-16}$$

$$= \int \frac{dx}{(x+4)^2 + 9}$$

a^2

$$= \frac{1}{3}\tan^{-1}\left(\frac{x+4}{3}\right) + C$$

$$\int \frac{dx}{\sqrt{6x-x^2}} = \int \frac{dx}{\sqrt{9-(x^2-6x+9)}}$$

$$= \int \frac{dx}{\sqrt{9-(x-3)^2}}$$

$$= \sin^{-1}\left(\frac{x-3}{3}\right) + C$$