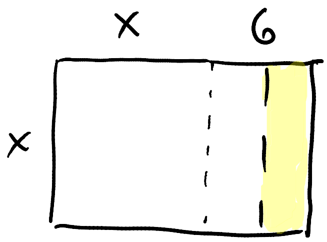
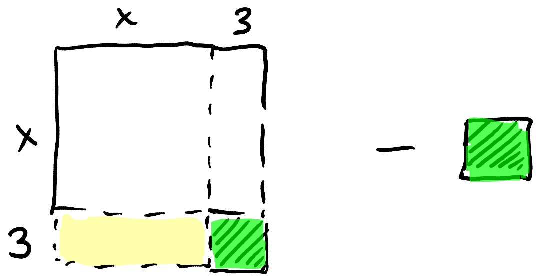
Section 28.6: continued

January-17-17
2:01 PM

idea behind completing the square

$$x^2 + 6x \quad \underset{\text{want}}{\Rightarrow} \quad (x + \_)^2 - \_^2$$





---

note: tomorrow's mini-quiz on 28.1 → 28.4 inclusive

(basic substitution)

---

back to 28.6:

more examples:

$$\int \frac{\overset{\text{five}}{5}\, p^2}{9 + p^6}\, dp$$

let $u = p^3$

$du = 3p^2\, dp$

$\frac{du}{3} = p^2\, dp$

$$\int \frac{5p^2}{9+p^6}\,dp$$

let $u = p^3$
$du = 3p^2\,dp$
$\frac{du}{3} = p^2\,dp$

$$= \int \frac{5}{3}\frac{du}{9+u^2}$$

$$= \frac{5}{3}\left(\frac{1}{3}\tan^{-1}\frac{u}{3}\right) + C$$

$$= \frac{5}{9}\tan^{-1}\frac{p^3}{3} + C$$

$$\int \frac{e^{3x}}{4+e^{6x}}\,dx \quad \text{vs} \quad \int \frac{e^{3x}}{4+e^{3x}}\,dx$$

let $u = e^{3x}$
$du = 3e^{3x}\,dx$

$$= \int \frac{1}{3}\frac{du}{4+u^2}$$

$$= \frac{1}{3}\frac{1}{2}\tan^{-1}\frac{u}{2} + C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{e^{3x}}{2}\right) + C$$

let $u = e^{3x}+4$
$du = 3e^{3x}\,dx$

$$= \int \frac{1}{3}\frac{du}{u}$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln\left(e^{3x}+4\right) + C$$

compare and contrast:

$$\int \frac{x\,dx}{} \quad \text{vs} \quad \int \frac{dx}{} \quad \text{vs} \quad \int \frac{x+1}{}\,dx$$

$$\int \frac{x\,dx}{x^2+1} \quad \text{vs} \quad \int \frac{dx}{x^2+1} \quad \text{vs} \quad \int \frac{x+1}{x^2+1}\,dx$$

substitute

let $u = x^2$

or $u = x^2+1$

arctan

break into

$$\int \left(\frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$\int_0^{\pi/4} \frac{\sec^2\theta}{\sqrt{4-\tan^2\theta}}\,d\theta$$

let $u = \tan\theta$

$du = \sec^2\theta\, d\theta$

if $u = \cancel{\tan^2\theta}$

$du = \cancel{2\tan\theta}\sec^2\theta\, d\theta$

$$= \int_{\theta=0}^{\theta=\pi/4} \frac{du}{\sqrt{4-u^2}}$$

$$= \sin^{-1}\left(\frac{u}{2}\right)\Bigg|_{\theta=0}^{\theta=\pi/4}$$

$$= \sin^{-1}\left(\frac{\tan\theta}{2}\right)\Bigg|_0^{\pi/4}$$

$$= \sin^{-1}\left(\frac{\tan \pi/4}{2}\right) - \sin^{-1}\left(\frac{\tan 0}{2}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin 0$$

$$= \frac{\pi}{6}$$