Section 28.7: Integration by Parts

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consider $\int x^2 \ln x \, dx$

note: this integral cannot be done by basic substitution in the way that $\int \frac{1}{x} \ln x \, dx$ could

recall the product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

in differential form:

$$d(uv) = u\,dv + v\,du$$

and if we integrate both sides, we get

$$\int d(uv) = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

solve for $\int u\,dv$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

on formula sheet

how does it work?

$$\int x^2 \ln x\,dx = \int \underbrace{\ln x}_{u}\ \underbrace{x^2 dx}_{dv}$$

let $u = \ln x$ $\qquad v = \frac{1}{3}x^3$ (ignore +C for now)

$du = \frac{1}{x}dx \qquad dv = x^2 dx$

now, write out the "parts" formula:

$$\int u\,dv = uv - \int v\,du$$

$$\int (\ln x)x^2 dx = (\ln x)(\tfrac{1}{3}x^3) - \underbrace{\int (\tfrac{1}{3}x^3)\frac{1}{x}dx}$$

why is this cool? because we can integrate it!

$$= \tfrac{1}{3}x^3 \ln x - \int \tfrac{1}{3}x^2 dx$$

$$= \tfrac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

each integral gives a constant, but we just lump them all together in our last step

example:

$$\int \underset{\uparrow \, u}{x} \; \underbrace{\cos 2x \; dx}_{dv}$$

$$\boxed{\begin{array}{ll} \text{let } u = x & v = \tfrac{1}{2}\sin 2x \\ du = dx & dv = \cos 2x \, dx \end{array}}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \cos 2x \, dx = x\left(\tfrac{1}{2}\sin 2x\right) - \int \tfrac{1}{2} \sin 2x \, dx$$

$$= \tfrac{1}{2} x \sin 2x - \tfrac{1}{2} \cdot \left(-\tfrac{1}{2}\cos 2x\right) + C$$

$$= \tfrac{1}{2} x \sin 2x + \tfrac{1}{4}\cos 2x + C$$

note $\frac{d}{dx}\left(\tfrac{1}{2} x \sin 2x + \tfrac{1}{4}\cos 2x + c\right)$

$= \frac{1}{2}\sin 2x + 2\cdot\frac{1}{2}x\cos 2x + \frac{1}{4}(-2\sin 2x)$

$= \frac{1}{2}\sin 2x + x\cos 2x - \frac{1}{2}\sin 2x$

$= x\cos 2x$

guidelines for u and dv:

idea: is to choose u so that du is simpler in form

and that dv is somethy you can integrate

basically two types

① $$\int \underset{u}{x^n} \underbrace{\left\{ \begin{matrix} e^{ax} \\ \sin ax \\ \cos ax \end{matrix} \right\} dx}_{dv}$$

note: you can integrate this dv

$$② \int x^n \begin{Bmatrix} \ln x \\ \tan^{-1} x \\ \sin^{-1} x \end{Bmatrix} dx$$

u so $dv = x^n dx$

nice memory aid:

let u = L I A T E

log, inverse trig, algebra, trig, exponent

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example:

$$\int \frac{x\,dx}{\sqrt{2x-1}}$$

let $u = x$

$du = dx$

$v = \frac{2}{2}(2x-1)^{1/2}$

$dv = (2x-1)^{-1/2}dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\,(2x-1)^{-1/2}dx = x(2x-1)^{1/2} - \int (2x-1)^{1/2}dx$$

$$= x(2x-1)^{1/2} - \frac{1}{2}\frac{2}{3}(2x-1)^{3/2} + C$$

$$= x(2x-1)^{1/2} - \frac{1}{3}(2x-1)^{3/2} + C$$