Section 28.7, cont'd

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integrate $\int x^2 \ln x \, dx$

let $u = \ln x$ $\quad v = \frac{1}{3}x^3$
$du = \frac{1}{x}dx$ $\quad dv = x^2 dx$

$$\int u \, dv = uv - \int v \, du$$
$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$$
$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

alternatively: tabular method

$\int x^2 \ln x \, dx$

	D	I	
$u =$	$\ln x$	x^2	$= dv$
	$\frac{1}{x}$	$\frac{1}{3}x^3$	

in the table method, diagonals are products and horizontals are integrals

$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x \; - \int \ldots$

$$\int x^2 \ln x \, d = \tfrac{1}{3}x^3 \ln x - \int \frac{1}{x} \cdot \frac{1}{3}x^3 \, dx$$

same thing as before

$$\int x^2 e^{2x} \, dx$$

if we use the usual method:

let $u = x^2$ $\quad v = \tfrac{1}{2}e^{2x}$

$du = 2x\,dx$ $\quad dv = e^{2x}\,dx$

$$= uv - \int v \, du$$

$$= \tfrac{1}{2}x^2 e^{2x} - \int \tfrac{1}{2} e^{2x} \, 2x \, dx$$

$$= \tfrac{1}{2} x^2 e^{2x} - \int x e^{2x} \, dx$$

let $u = x$ $\quad v = \tfrac{1}{2}e^{2x}$

$du = dx$ $\quad dv = e^{2x}\,dx$

$$= \tfrac{1}{2}x^2 e^{2x} - \left[uv - \int v \, du \right]$$

$$= \tfrac{1}{2}x^2e^{2x} - \tfrac{1}{2}xe^{2x} + \int \tfrac{1}{2}e^{2x}\,dx$$

$$= \tfrac{1}{2}x^2e^{2x} - \tfrac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

<u>but</u> same integral with table method:

$$\int x^2e^{2x}\,dx$$

D		I
x^2	$+$	e^{2x}
$2x$	$-$	$\frac{1}{2}e^{2x}$
2	$+$	$\frac{1}{4}e^{2x}$
0		$\frac{1}{8}e^{2x}$

$$= \frac{1}{2}x^2e^{2x} - 2x\left(\frac{1}{4}e^{2x}\right) + 2\left(\frac{1}{8}e^{2x}\right) + C$$

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

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what about $\int e^x \cos x\,dx$?

D	I

D		I
$\cos x$	$+$	e^x
$-\sin x$	$-$	e^x
$-\cos x$	$+$	e^x

$$\int e^x \cos x \, dx = e^x \cos x - \int (-\sin x) e^x \, dx$$

$$= e^x \cos x - (-\sin x) e^x + \int (-\cos x) e^x \, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$+ \int e^x \cos x \, dx \qquad\qquad + \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \tfrac{1}{2} e^x \cos x + \tfrac{1}{2} e^x \sin x + C$$

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special cases

$$\int \sin^{-1} x \, dx$$

let $u = \sin^{-1} x$ $\qquad v = x$

$du = \dfrac{1}{\sqrt{1-x^2}} dx$ $\qquad dv = dx$

$$= uv - \int v\,du$$

$$= x\sin^{-1}x - \int \frac{x\,dx}{\sqrt{1-x^2}}$$

let $w = 1-x^2$
$dw = -2x\,dx$

$$= x\sin^{-1}x - \int -\frac{1}{2} w^{-1/2}\,dw$$

$$= x\sin^{-1}x + \frac{1}{2}\,\frac{w^{1/2}}{1/2} + C$$

$$= x\sin^{-1}x + (1-x^2)^{1/2} + C$$

check:

$$\frac{d}{dx}\left(x\sin^{-1}x + \sqrt{1-x^2}\right)$$

$$= x\cdot\frac{1}{\sqrt{1-x^2}} + 1\cdot\sin^{-1}x + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x$$