

Section 28.9: cont'd

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mini-quiz Wed:

on inverse trig including completing square

or integration by parts

or partial fractions incl long division

suppose you have $\int \frac{x^2 + 5x + 4}{(x-1)(x-3)(x+2)} dx$

$$\frac{x^2 + 5x + 4}{(x-1)(x-3)(x+2)} = \frac{A}{x-1} + \frac{B}{x-3} + \frac{D}{x+2}$$

$$x^2 + 5x + 4 = A(x-3)(x+2) + B(x-1)(x+2) + D(x-1)(x-3)$$

let $x = 3$:

$$(3)^2 + 5(3) + 4 = A \cdot 0 + B(2)(5) + 0 \cdot 0$$

etc

one last full example:

evaluate $\int \frac{3x^2 - 13x - 6}{x^2 - 6x} dx$

$$x^2 - 6x \quad \begin{array}{r} 3 \\ \hline 3x^2 - 13x - 6 \\ 3x^2 - 18x \\ \hline 5x - 6 \end{array}$$

$$\frac{3x^2 - 13x - 6}{x^2 - 6x} = 3 + \frac{5x - 6}{x^2 - 6x}$$

partial fractions:

$$\frac{5x - 6}{x(x - 6)} = \frac{A}{x} + \frac{B}{x - 6}$$

$$5x - 6 = A(x - 6) + Bx$$

$$\begin{array}{l} \text{when } x = 0, \\ x = 6 \end{array}$$

$$-6 = -6A$$

$$24 = 6B$$

$$\text{so } A = 1$$

$$B = 4$$

$$\begin{aligned} \int \frac{3x^2 - 13x - 6}{x^2 - 6x} dx &= \int \left[3 + \frac{1}{x} + \frac{4}{x - 6} \right] dx \\ &= 3x + \ln |x| + 4 \ln |x - 6| + C \end{aligned}$$

note:

$$\int \frac{dx}{x^2 + 5x + 4}$$



denom factors,
so use
partial fractions

vs

$$\int \frac{dx}{x^2 + 4x + 5}$$



denom does
not factor,
but completing
the square
gives

$$\int \frac{dx}{(x+2)^2 + 1}$$

which is arctan

note: omit 28.10