Section 29.3: Partial Derivatives

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we've been doing "calculus of a single variable" up until now

what happens when we try to take the derivative of a function of two or more variables?

→ these derivatives are called

"partial derivatives"

and are written:

$$\frac{\partial f}{\partial x}$$

∂ is like a backwards six, not a "dee"

$$\frac{\partial f}{\partial x} \quad \text{and} \quad \frac{\partial f}{\partial y}$$

other notations:

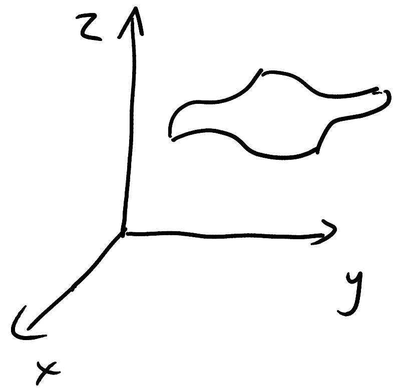
$$f_x(x,y) \quad , \quad f_y(x,y)$$

$$\frac{\partial}{\partial x} f(x,y) \quad , \quad \frac{\partial}{\partial y} f(x,y)$$

↑ ↑

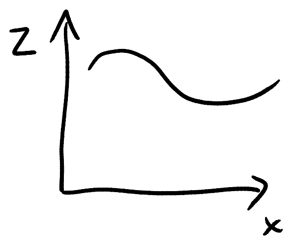
note: <u>must</u> specify which variable you are differentiating with respect to!

main idea:



$\Leftarrow$ some curved surface

- then take a slice parallel to the x-z plane



the slope of this line is $\frac{\partial f}{\partial x}$

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so, how to do you calculate partial derivatives?

- treat the other variable as if it were a constant and differentiate as usual

example: find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = x^2 + 2xy$

$$\frac{\partial f}{\partial x} = 2x + 2y$$

$$\frac{\partial f}{\partial x} = 6x + 2y$$

$$\frac{\partial f}{\partial y} = 0 + 2x = 2x$$

note: the actual full definition of partial derivative:

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

but we won't get into that

example:

find $\left.\frac{\partial z}{\partial y}\right|_{(3, \pi/2, 9)}$ for $z = x^2 \cos 4y$

answer: $\frac{\partial z}{\partial y} = -4x^2 \sin 4y$

$$\left.\frac{\partial z}{\partial y}\right|_{(3, \pi/2, 9)} = -4(3)^2 \sin\left(4 \cdot \pi/2\right)$$

$$= 0$$

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let $z = e^y \ln x$

a) find both first partial derivatives

$$\frac{\partial z}{\partial x} = \frac{e^y}{x} \qquad \frac{\partial z}{\partial y} = e^y \ln x$$

b) find all four second partial derivatives

so, what is $\frac{\partial^2 z}{\partial x^2}$? it's what you get when you differentiate z with respect to x twice

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{e^y}{x}\right)$$

$$= -\frac{e^y}{x^2}$$

but there's also

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial y}\left(e^y \ln x\right)$$

$$= e^y \ln x$$

but wait! there's more!

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(e^y \ln x\right)$$

$$= \frac{e^y}{x}$$

and $$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

same!

$$= \frac{\partial}{\partial y}\left(\frac{e^y}{x}\right)$$

$$= \frac{e^y}{x}$$

note: $$\underbrace{\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}}$$

always true for all $z(x,y)$

if z is continuous