section 29.4: cont'd

Thursday, January 26, 2017 1:30 PM

example: evaluate

$$\int_0^{\pi} \int_1^3 x \sin y \, dx \, dy$$

$$= \int_0^{\pi} \left[\frac{x^2}{2} \sin y \Big|_1^3 \right] dy$$

$$= \int_0^{\pi} \left[\tfrac{9}{2} \sin y - \tfrac{1}{2} \sin y \right] dy$$

$$= \int_0^{\pi} 4 \sin y \, dy$$

$$= -4 \cos y \Big|_0^{\pi}$$

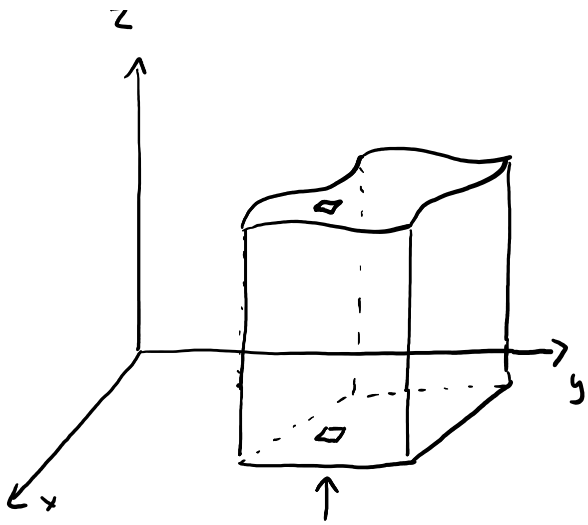
$$= -4 \cos \pi + 4 \cos 0$$

$$= -4(-1) + 4(1)$$

$$= 8$$

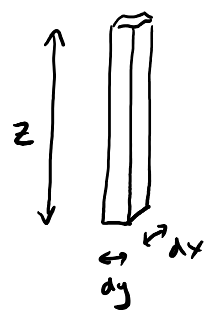
applications: volume under a 3D surface

consider a surface $z = f(x, y)$



what is the volume underneath this 3D surface?

consider the little area element dA in the x-y plane



$$dV = z\, dA$$
$$= z\, dx\, dy$$
$$= f(x,y)\, dx\, dy$$

or could swap to get dy dx

in general

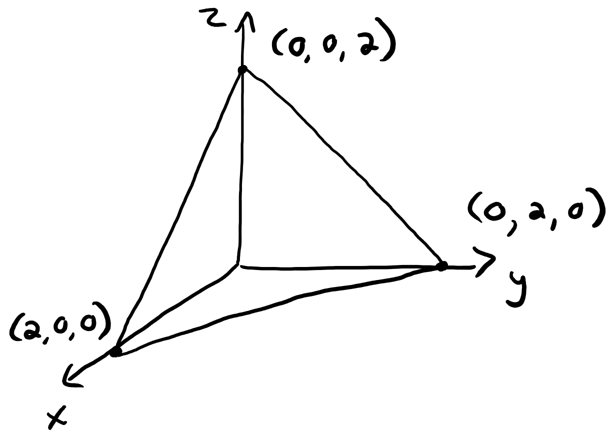
$$V = \int_V dV$$

$$= \int_V f(x,y)\, dA$$

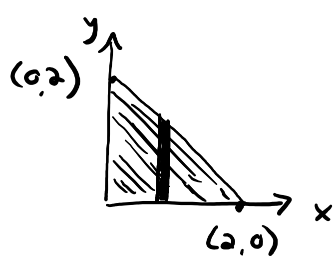
$$= \int_V f(x,y)\, \underbrace{dx\, dy}_{\text{or } dy\, dx}$$

so, let's do an example, including how to set up the limits

example: Find the volume under the plane $x + y + z = 2$ in the first octant



step 1: draw a picture of the x-y plane (z=0)
$x + y = 2$



note: $y = 2 - x$

step 2: take a slice in x-y plane to set up the limits

hard limits $0 \le x \le 2$
soft limits $0 \le y \le$ line
$0 \le y \le 2 - x$

step 3: set up integral:

$$V = \int_V f(x,y)\, dA$$

$$= \int_0^2 \int_0^{2-x} f(x,y) \quad dy\, dx$$

but what's this? it's z

$$x + y + z = 2$$

$$z = 2 - x - y$$

$$V = \int_0^2 \int_0^{2-x} (2 - x - y)\, dy\, dx$$