Section 31.2: Separation of variable

Friday, February 3, 2017 2:38 PM

so, we'll now learn some methods for how to solve DEs

first up: separation of variables

example: solve

$$2\frac{dy}{dx} = y\frac{(x+1)}{x}$$

$$2\,dy = y\frac{(x+1)}{x}\,dx$$

(why? $\frac{dy}{dx}\,dx = dy$)

$$2\frac{dy}{y} = \frac{x+1}{x}\,dx$$

← separated (all x's on one side, all y's on the other)

$$\int \frac{2\,dy}{y} = \int \frac{x+1}{x}\,dx$$

$$2\ln|y| = \int \left(1+\frac{1}{x}\right)dx$$

$$2\ln|y| = x + \ln|x| + C$$

note: the textbook isn't very strict about absolute values here so the solution to our DE is

$$\underbrace{2 \ln y}_{\text{perfectly acceptable implicit solution}} = x + \ln x + C$$

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consider a DE of first order and first degree:

$$\frac{dy}{dx} = f(x, y)$$

IF we can rewrite it into:

$$\underbrace{A(x)\,dx}_{\text{contains only } x} = \underbrace{B(y)\,dy}_{\text{contains only } y}$$

then we just integrate both sides to solve.

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example: solve $y\,dt + t\,dy = 0$

$$\frac{y\,dt}{yt} = -\frac{t\,dy}{yt}$$

$$\frac{dt}{t} = -\frac{dy}{y}$$

$$\int \frac{dt}{t} = \int -\frac{dy}{y}$$

perfectly acceptable answer #1 → $\ln t = -\ln y + C$

implicit

but what if I asked for an explicit sol'n?

$$\ln t = -\ln y + C$$

$$\ln t + \ln y = C$$

$$\ln(ty) = C$$

$$ty = e^C$$

$$y = \frac{e^C}{t}$$

but note: e^C is just another constant

say $C_1 = e^C$

so $\boxed{y = \frac{C_1}{t}}$ explicit

example:

$$\sqrt{x}\, y^2\, dy + \frac{x^3}{y}\, dx = 0$$

$$\sqrt{x}\, y^2\, dy = -\frac{x^3\, dx}{y}$$

$$y^3\, dy = -\frac{x^3}{\sqrt{x}}\, dx$$

$$\int y^3\, dy = \int -x^{5/2}\, dx$$

$$\frac{y^4}{4} = -\frac{2}{7}x^{7/2} + C$$

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example:

$$dm - m^2\, dt = 9\, dt$$

$$dm = m^2\, dt + 9\, dt$$

$$dm = (m^2 + 9)\, dt$$

$$\int \frac{dm}{m^2+9} = \int dt$$

$$\frac{1}{3}\tan^{-1}\frac{m}{3} = t + C$$