Section 31.2: cont'd

Monday, February 6, 2017 1:05 PM

example: solve

$$\sin x \sec y \, dx = dy$$

$$\sin x \, dx = \frac{dy}{\sec y}$$

$$\int \sin x \, dx = \int \cos y \, dy$$

implicit solution ⇒
(not solved for y)

$$-\cos x = \sin y + C$$

---

example: solve

$$x y y' + \sqrt{1+y^2} = 0$$

$$x y \frac{dy}{dx} dx + \sqrt{1+y^2} \, dx = 0$$

$$x y \, dy + \sqrt{1+y^2} \, dx = 0$$

$$x y \, dy = -\sqrt{1+y^2} \, dx$$

$$\int \frac{y \, dy}{\sqrt{1+y^2}} = \int -\frac{dx}{x}$$

let $u = 1+y^2$
$du = 2y \, dy$

$$\int u^{-1/2} \frac{du}{2} = \int \frac{-dx}{x}$$

$$u^{1/2} = -\ln|x| + C$$

$$\frac{u^{1/2}}{2 \cdot 1/2} = -\ln|x| + C$$

$$\sqrt{1+y^2} = -\ln|x| + C$$

---

example: solve $\frac{dr}{dt} = \sec r$, given that $t=0$ when $r=0$

initial value problem

$$\frac{dr}{\sec r} = dt$$

$$\int \cos r \, dr = \int dt$$

general solution (constant) → $\sin r = t + C$

at $r=0$, $t=0$

$$\sin 0 = 0 + C$$

$$C = 0$$

$$\boxed{\sin r = t}$$

---

example: solve the following DE, given that $y = \pi/2$ when $x = 0$

$$\frac{2y \cos y \, dy - \sin y \, dy}{y \sin y} = \frac{y \sin y \, dx}{y \sin y}$$

$$2\frac{\cos y}{\sin y}\,dy - \frac{dy}{y} = dx$$

$$\int\left(2\cot y - \frac{1}{y}\right)dy = \int dx$$

$$2\ln|\sin y| - \ln|y| = x + C$$ ← general solution

but when $x = 0$, $y = \pi/2$

$$2\ln|\sin \pi/2| - \ln \pi/2 = 0 + C$$

$$2\ln 1^{\,0} - \ln \pi/2 = C$$

$$C = -\ln \pi/2$$

so $$\boxed{2\ln|\sin y| - \ln|y| = x - \ln \pi/2}$$

particular solution

note that $x = 2\ln|\sin y| - \ln|y| + \ln \pi/2$

$$= \ln \sin^2 y - \ln y + \ln \pi/2$$

$$\boxed{= \ln \frac{\pi \sin^2 y}{2y}}$$