Review :

Wednesday, February 8, 2017 1:36 PM

solve $e^{2x}\,dy + e^{x}\,dx = 4\,dx$

$$e^{2x}\,dy = 4\,dx - e^{x}\,dx$$

$$dy = \frac{4\,dx}{e^{2x}} - \frac{e^{x}\,dx}{e^{2x}}$$

$$\int dy = \int 4e^{-2x}\,dx - \int e^{-x}\,dx \qquad \text{or} \quad = \int \left(4e^{-2x} - e^{-x}\right) dx$$

$$y = \frac{4e^{-2x}}{-2} - \frac{e^{-x}}{-1} + C$$

$$\boxed{y = -2e^{-2x} + e^{-x} + C}$$

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solve : $y^2 e^t + (e^t + 4)\,y' = 0$

$$y^2 e^t + (e^t + 4)\frac{dy}{dt} = 0$$

$$y^2 e^t = -(e^t + 4)\frac{dy}{dt}$$

$$y^2 e^t\,dt = -(e^t + 4)\,dy$$

$$\frac{e^t\,dt}{e^t + 4} = -\frac{dy}{y^2}$$

let $u = e^t + 4$

$du = e^t\,dt$

$$\int \frac{du}{u} = \int -y^{-2}\,dy$$

$$\ln|u| = -\frac{y^{-1}}{-1} + C$$

$$\ln(e^t + 4) = y^{-1} + C$$

now write as an explicit solution:

$$\ln(e^t+4) = \frac{1}{y} + C$$

$$\ln(e^t+4) - C = \frac{1}{y}$$

$$y = \frac{1}{\ln(e^t+4) - C}$$

what if when $t=0$, $y=3$?

$$\ln(e^t+4) - C = \frac{1}{y}$$

$$\ln(e^0+4) - C = \frac{1}{3}$$

$$\ln 5 - C = \frac{1}{3}$$

$$\ln 5 - 1/3 = C$$

then $$y = \frac{1}{\ln(e^t+4) - \ln 5 + 1/3}$$

$$= \frac{3}{3\ln(e^t+4) - 3\ln 5 + 1}$$

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evaluate $$\int_0^4 \int_0^{\sqrt{y}} (x-y)\,dx\,dy$$

$$= \int_0^4 \left[\left(\frac{x^2}{2} - yx\right)\Big|_{x=0}^{x=\sqrt{y}} \right] dy$$

$$= \int_0^4 \left[\frac{(\sqrt{y})^2}{2} - y\sqrt{y} \right] dy$$

$$= \int_0^4 \left(\frac{y}{2} - y^{3/2} \right) dy$$

$$= \left(\frac{y^2}{4} - \frac{2}{5} y^{5/2} \right)\Big|_0^4$$

$= \frac{16}{4} - \frac{2}{5} 4^{5/2}$

$= 4 - 32/5$

$= -12/5$ or -2.4