Section 31.4: cont'd

Thursday, February 9, 2017 1:48 PM

two useful properties of logs:

$$\ln x^4 = 4 \ln x$$

$$e^{\ln x} = x$$

$$e^{\ln x} = y$$
$$\ln e^{\ln x} = \ln y$$
$$\ln x = \ln y$$
$$x = y$$

please note:
it must be in exactly this form

$$e^{2\ln x} = e^{\ln x^2} = x^2$$

$$e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} \text{ or } \frac{1}{x}$$

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solve $xy' - 4y = x^5 e^{2x}$

give an explicit solution.

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$$

$$y' - \frac{4}{x}y = x^4 e^{2x}$$

$$IF = e^{\int P(x)\,dx} = e^{\int -\frac{4}{x}\,dx}$$

$$= e^{-4\ln x}$$

$$= e^{\ln x^{-4}}$$

$$= x^{-4}$$

$$x^{-4}\,y' - 4y\,x^{-5} = x^{-4}x^{4}e^{2x}$$

$$x^{-4}\frac{dy}{dx} - 4y\,x^{-5} = e^{2x}$$

$$\frac{d}{dx}\left(y\,x^{-4}\right) = e^{2x}$$

$$\int d\left(y\,x^{-4}\right) = \int e^{2x}\,dx$$

$$y\,x^{-4} = \frac{e^{2x}}{2} + C$$

$$y = \frac{1}{2}x^{4}e^{2x} + Cx^{4}$$

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solve: $$y' + y\tan x = -\sin x$$

$$\frac{dy}{dx} + (\tan x)\,y = -\sin x$$

$$IF = e^{\int P(x)\,dx} = e^{\int \tan x\,dx}$$

$$= e^{-\ln(\cos x)}$$

$$= e^{\ln(\cos x)^{-1}}$$

$$= (\cos x)^{-1} = \frac{1}{\cos x} = \sec x$$

$$\sec x \frac{dy}{dx} + (\tan x \sec x)\, y = -\sin x \sec x$$

$$\frac{d}{dx}(y \sec x) = -\sin x \sec x$$

$$\int d(y \sec x) = \int -\frac{\sin x}{\cos x}\, dx = \int -\tan x \, dx$$

$$y \sec x = -\ln(\sec x) + C$$

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solve, giving an explicit solution:

$$x \frac{dy}{dx} = 4y + 4x^6$$

$$\frac{dy}{dx} - \frac{4}{x} y = 4x^5$$

$$IF = e^{\int p(x)\,dx} = e^{\int -\frac{4}{x}\,dx} = e^{-4\ln x}$$

$$= e^{\ln x^{-4}}$$

$$= x^{-4}$$

$$x^{-4}\frac{dy}{dx} - 4x^{-5}y = 4x$$

$$\frac{d}{dx}\left(y x^{-4}\right) = 4x$$

$$\int d\left(y x^{-4}\right) = \int 4x\,dx$$

$$y x^{-4} = 2x^2 + C$$

$$y = 2x^6 + Cx^4$$