Section 3.6: Applications for first-order DEs

Friday, February 10, 2017 2:32 PM

note: we will omit the electrical circuit applications

why do we care about solving DEs? because they show up in so many applications!

word problems on handout

(1) rate of growth of population: $\frac{dP}{dt}$

a) $$\frac{dP}{dt} = kP$$ where k = some constant

this is a differential equation we can solve!

b) this one separates:

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln|P| = kt + C$$

$$\ln P = kt + C$$

$$P = e^{kt+c}$$

$$P = e^{kt}\, e^{c}$$

e^{c} is a constant

$e^{c} = C_1$

$$P = C_1 e^{kt}$$

but at $t=0$, $P = P_0$

$$P_0 = C e^{k \cdot 0}$$

$$P_0 = C$$

so $$\boxed{P = P_0 e^{kt}}$$

c) doubling time $\equiv$ time it takes the population to double (increase by a factor of 2)

finding k: 5% $\rightarrow$ 100% in 2 years

$$P = P_0 e^{kt}$$

$$\frac{P}{P} = \frac{0.05 P}{P} e^{k \cdot 2} \qquad (P\text{'s cancel})$$

$$1 = 0.05\, e^{2k}$$

$$20 = e^{2k}$$ hint: get rid

$$20 = e^{2k}$$

hint: get rid of coeff first!

$$\ln 20 = \ln e^{2k}$$

$$\ln 20 = 2k$$

$$k = \frac{\ln 20}{2}$$

now find doubling time:

$$P = P_0 e^{kt}$$

$$2\cancel{P_0} = \cancel{P_0} e^{kt}$$

$$2 = e^{kt}$$

$$\ln 2 = kt$$

$$t = \frac{\ln 2}{k} = \frac{\ln 2}{\frac{1}{2}\ln 20}$$

$$\approx 0.4627 \text{ years}$$

so $\boxed{0.46 \text{ years}}$ or $\boxed{0.5 \text{ years}}$

---

② a) $$\frac{dN}{dt} = -kN$$

note: $\frac{dN}{dt} = kN$

(2) a) $$\frac{dN}{dt} = -kN$$

note: $\frac{dN}{dt} = kN$ also acceptable

b) $$\frac{dN}{N} = -k\,dt$$

$$\ln N = -kt + C$$

$$N = e^{-kt+C}$$

$$= C_1 e^{-kt}$$

at $t=0$, $N = N_0$

$$N_0 = C_1 e^0 \qquad \text{so} \qquad C_1 = N_0$$

$$\boxed{N = N_0 e^{-kt}}$$