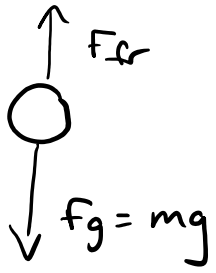
Section 31.6:

Monday, February 20, 2017 1:08 PM

(4) falling object

a)



$$\sum \vec{F} = m\vec{a}$$

$$mg - kv = ma$$

$$f_{fr} = kv$$

b) recall: $a = \frac{dv}{dt}$

so

$$\boxed{mg - kv = m\frac{dv}{dt}}$$

note: m, g, and k are constants

c) $$m\frac{dv}{dt} = mg - kv$$

$$\frac{dv}{dt} = g - \frac{k}{m}v$$

$$\int \frac{dv}{g - \frac{k}{m}v} = \int dt$$

separable

let $u = g - \frac{k}{m} v$

$du = -\frac{k}{m} dv$

$-\frac{m}{k} du = dv$

$$\int \frac{-\frac{m}{k} du}{u} = \int dt$$

$$-\frac{m}{k} \ln |u| = t + C$$

$$-\frac{m}{k} \ln \left| g - \frac{k}{m} v \right| = t + C$$

$$-\frac{m}{k} \ln \left(g - \frac{k}{m} v \right) = t + C$$

so now solve for v:

$$\ln \left(g - \frac{k}{m} v \right) = -\frac{k}{m} t + -\frac{k}{m} C$$

$$g - \frac{k}{m} v = e^{-\frac{k}{m} t - \frac{k}{m} C}$$

$$g - \frac{k}{m} v = e^{-\frac{k}{m} t} e^{-\frac{k}{m} C}$$

" get another constant C_1

$$g - \frac{k}{m} v = C_1 e^{-k/m\, t}$$

$$-\frac{k}{m}v = C_1 e^{-k/m\,t} - g$$

$$v = -\frac{C_1 m}{k} e^{-k/m\,t} + \frac{gm}{k}$$

c) starting from rest: at $t = 0$, $v = 0$

$$0 = -\frac{C_1 m}{k} e^{0} + \frac{gm}{k}$$

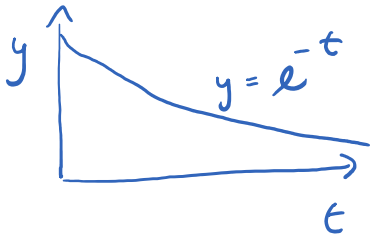
$$\frac{mC_1}{k} = \frac{gm}{k}$$

$$C_1 = g$$

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

d) what happens as $t \to \infty$?

recall



so as $t \to \infty$, $e^{-t} \to 0$

$$\lim_{t\to\infty} v = \lim_{t\to\infty} \frac{mg}{k}\left(1 - e^{-km/t}\right)$$

$= \frac{mg}{k}$ terminal velocity

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(3) variable: T

rate of change: $\frac{dT}{dt}$

$$\frac{dT}{dt} = k(T - T_{room})$$

separable

$$\int \frac{dT}{T - T_{room}} = \int k\,dt$$

$$\ln(T - T_{room}) = kt + C$$

note:

$a \propto b$

$a = (\text{some constant})\, b$