Section 31.6: cont'd

Tuesday, February 21, 2017 2:01 PM

③ a) variable: T

$$\frac{dT}{dt} = k(T - T_{room})$$

↑ if you like, can put a negative sign in front (since T is decreasing)

b) solve the DE:

$$\int \frac{dT}{T - T_{room}} = \int k\,dt$$

$$\ln(T - T_{room}) = kt + C$$

$$T - T_{room} = e^{kt + C}$$

$$T = e^{kt} e^{C} + T_{room}$$

$$= C_1 e^{kt} + T_{room}$$

initial condition: at $t = 0$, $T = T_{init}$

$$T_{init} = C_1 e^{k \cdot 0} + T_{room}$$

$$C_1 = T_{init} - T_{room}$$

so $T = (T_{init} - T_{room}) e^{kt} + T_{room}$

c) it doesn't (though if you'd used $-k$ in part (a), it's nice to use $+k$ here because T is increasing)

d) $T_{init} = -200°C$, $T_{room} = 20°C$

it takes 1 day to have $T = -100°C$ ← use to find k

it takes ? days to get to $T = 0°C$

$$-100 = (-200 - 20)e^{k \cdot 1} + 20$$

$$-120 = -220e^{k}$$

$$\frac{120}{220} = e^{k}$$

$$\ln \frac{120}{220} = k$$

so $k \approx -0.606136$

(note: do not round constants in exponents - keep extra digits)

now, find t:

$$T = (T_{init} - T_{room})e^{kt} + T_{room}$$

$$0 = (-200 - 20)e^{-0.606136t} + 20$$

$$-20 = -220e^{-0.606136t}$$

$$\frac{20}{220} = e^{-0.606136t}$$

$$\ln \frac{20}{220} = -0.606136t$$

220

$$t = \frac{\ln \frac{20}{220}}{-0.606136} \approx 3.956 \text{ days}$$

$$\approx 4 \text{ days}$$

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note for the unwary: what happens if you are not careful about solving for k?

$$-120 = -220\, e^{k}$$

$$120 = 220\, e^{k}$$

$$\ln 120 = \ln (220\, e^{k})$$

$$\ln 120 = \ln 220 + \ln e^{k}$$

$$\ln 120 - \ln 220 = \ln e^{k}$$

$$\ln \frac{120}{220} = k$$

but easier to get rid of coefficient first!

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in general, if your variable is "y"

$$\frac{dy}{dt} = \text{increase in } y - \text{decrease in } y$$

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note: omit the "tank / brine"

questions