Section 31.7 / 31.8: cont'd

Wednesday, February 22, 2017 1:49 PM

example: follow procedure on handout

solve $y'' - 2y' - 8y = 0$

step ① write the auxiliary equation

$m^2 - 2m - 8 = 0$

solve for m: $(m-4)(m+2) = 0$ factor or use quad. formula

$m = 4, -2$

note: 2 distinct real roots

step ② write solution

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

so $y = c_1 e^{4x} + c_2 e^{-2x}$

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quadratic formula:

$0 = ax^2 + bx + c$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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check: is this solution <u>truly</u> a soln to the DE?

$y = c_1 e^{4x} + c_2 e^{-2x}$

$$y' = 4C_1 e^{4x} - 2C_2 e^{-2x}$$

$$y'' = 16C_1 e^{4x} + 4C_2 e^{-2x}$$

$$y'' - 2y' - 8y = 0$$

$$(16C_1 e^{4x} + 4C_2 e^{-2x}) - 2(4C_1 e^{4x} - 2C_2 e^{-2x}) - 8(C_1 e^{4x} + C_2 e^{-2x}) = 0$$

$$16C_1 e^{4x} + 4C_2 e^{-2x} - 8C_1 e^{4x} + 4C_2 e^{-2x} - 8C_1 e^{4x} - 8C_2 e^{-2x} = 0$$

$$0 = 0 \quad \checkmark$$

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example: solve $y'' - 5y' + 4y = 0$

$$m^2 - 5m + 4 = 0$$

$$(m-4)(m-1) = 0$$

$$m = 4, 1$$

$$y = C_1 e^{4x} + C_2 e^{x}$$

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example: solve $y'' - 2y' - 6y = 0$

$$m^2 - 2m - 6 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{2}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

$= \frac{2 \pm 2\sqrt{7}}{2}$

$= 1 \pm \sqrt{7}$

$y = C_1 e^{(1+\sqrt{7})x} + C_2 e^{(1-\sqrt{7})x}$

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example: solve $y''' - 4y'' - 21y' = 0$

$m^3 - 4m^2 - 21m = 0$

$m(m^2 - 4m - 21) = 0$

$m(m-7)(m+3) = 0$

$m = 0, 7, -3$

$y = C_1 e^{0x} + C_2 e^{7x} + C_3 e^{-3x}$

$y = C_1 + C_2 e^{7x} + C_3 e^{-3x}$

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solve the following DE, given that when $x = 0$, $y = 1$ and $y' = 2$.

$y'' + y' = 0$

$m^2 + m = 0$

$m(m+1) = 0$

$m = 0, -1$

$y = e^{0x} \quad e^{-x}$

$$y = c_1 e^{0x} + c_2 e^{-x}$$

$$y = c_1 + c_2 e^{-x} \quad \leftarrow \text{general solution}$$

now use initial conditions:

when $x = 0$, $y = 1$:

$$y = c_1 + c_2 e^{-x}$$
$$1 = c_1 + c_2$$

$x = 0$, $y' = 2$

$$y' = -c_2 e^{-x}$$
$$2 = -c_2$$

and plug into $1 = c_1 + c_2$

$$c_1 = 3$$

$$y = 3 - 2e^{-x}$$