Tutorial

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Assume that sugar dissolves at a rate proportional to the undissolved amount. If there are initially 525 g of sugar and 225 remain after 4.00 min how long does it take to dissolve 375 g?

let A = undissolved amount

$$\frac{dA}{dt} = kA \qquad \text{DE}$$

$$\int \frac{dA}{A} = \int k\,dt$$

$$\ln A = kt + C$$

$$A = e^{kt+C} = e^{kt} e^{C} = C_1 e^{kt}$$

at $t=0$, $A=525$

$$525 = C_1 e^{k \cdot 0} \qquad \text{so} \quad C_1 = 525$$

at $t=4$, $A=225$

$$225 = 525\, e^{k \cdot 4}$$

$$\frac{225}{525} = e^{4k}$$

$$\ln \frac{225}{525} = 4k$$

$\frac{\quad}{525} = \quad$

$k = \frac{1}{4} \ln\left(\frac{225}{525}\right)$

when $t = ?$, how long does it take to dissolve 375 g?

dissolved = 375 g

undissolved = 525 − 375

= 150

$A = 525\, e^{kt}$

$150 = 525\, e^{kt}$

$\frac{150}{525} = e^{kt}$

$\ln\left(\frac{150}{525}\right) = kt$

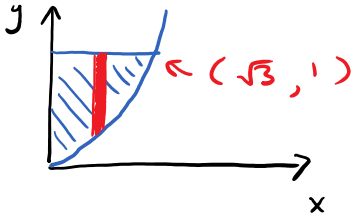
$t = \frac{\ln\left(\frac{150}{525}\right)}{k} = \frac{\ln\left(\frac{150}{525}\right)}{\frac{1}{4}\ln\left(\frac{225}{525}\right)}$

$= 5.91416 \text{ min}$

$= 5.9 \text{ min}$

Find the volume in the first octant that is under the surface $z = 4 - x^2 - y^2$ and is between the parabola $x^2 = 3y$ and the plane $y = 1$.

hint:



$x^2 = 3y$

$x^2 = 3$ when $y = 1$

$x = \sqrt{3}$

$$V = \int_V z \, dx \, dy \qquad (\text{or } dy \, dx)$$

$$0 \le x \le \sqrt{3} \qquad \leftarrow \text{outside}$$
$$x^2/3 \le y \le 1$$

$$V = \int_0^{\sqrt{3}} \int_{x^2/3}^{1} z \, dy \, dx$$

$$= \int_0^{\sqrt{3}} \int_{x^2/3}^{1} (4 - x^2 - y^2) \, dy \, dx$$