Section 31.7/8 cont'd

Monday, February 27, 2017 12:56 PM

recall: complex numbers

solve $m^2 - 6m + 13 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 - 4(13)}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2}$$

$$\left[\sqrt{-16} = \sqrt{16}\sqrt{-1} = 4i\right]$$

$$= 3 \pm 2i$$

so how do you use this to solve

$$y'' - 6y' + 13y = 0$$

$m^2 - 6m + 13 = 0$

↓ solve

$m = 3 \pm 2i$

$m = a + bi$

$$y = e^{ax}(C_1 \cos bx + C_2 \sin bx)$$

$$= e^{3x}(C_1 \cos 2x + C_2 \sin 2x)$$

solve $y'' - 6y' + 34y = 0$

$m^2 - 6m + 34 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 - 136}}{2}$$

$$= \frac{6 \pm \sqrt{-100}}{2}$$

$$= \frac{6 \pm 10i}{2}$$

$$= 3 \pm 5i$$

$$= a \pm bi$$

$$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$$

$$= e^{3x}(c_1 \cos 5x + c_2 \sin 5x)$$

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short explanation: could use answer for step 2 of handout

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$= c_1 e^{(3+5i)x} + c_2 e^{(3-5i)x}$$

a fine answer, but kind of annoying to deal

with complex numbers in exponents

$$e^{ix} = \cos x + i \sin x$$

longer explanation:

$$y = C_1 e^{(3+5i)x} + C_2 e^{(3-5i)x}$$

$$= C_1 e^{3x} e^{5ix} + C_2 e^{3x} e^{-5ix}$$

$$= e^{3x}\left(C_1 e^{5ix} + C_2 e^{-5ix}\right)$$

$$= e^{3x}\left(C_1 \cos 5x + C_1 i \sin 5x + C_2 \cos 5x - C_2 i \sin 5x\right)$$

$$= e^{3x}\left(C_1 \cos 5x + C_2 \cos 5x + C_1 i \sin 5x - C_2 i \sin 5x\right)$$

$$= e^{3x}\left[\underbrace{(C_1 + C_2)}_{\text{new constant } A} \cos 5x + \underbrace{(C_1 - C_2) i}_{\text{new constant } B} \sin 5x\right]$$

$$= e^{3x}\left[A \cos 5x + B \sin 5x\right]$$

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solve the following DE, given that when $x = 0$, $y = 0$, and $y' = 24$

$$y'' + 36y = 0$$

$$m^2 + 36 = 0$$

$$m^2 = -36$$

$$m = \pm\sqrt{-36}$$

$$= \pm 6i$$

$$= a \pm bi \quad \text{where } a = 0$$

$b = 6$

$$y = e^{ax}(C_1 \cos bx + C_2 \sin bx)$$

$$y = e^{0x}(C_1 \cos 6x + C_2 \sin 6x)$$

$$\boxed{y = C_1 \cos 6x + C_2 \sin 6x}$$ general

initial conditions:

when $x = 0$, $y = 0$

$$y = C_1 \cos 6x + C_2 \sin 6x$$
$$0 = C_1 \cos 0 + C_2 \sin 0$$
$$0 = C_1$$

$$\therefore y = C_2 \sin 6x$$

when $x = 0$, $y' = 24$

$$y' = 6C_2 \cos 6x$$
$$24 = 6C_2 \cos 0$$
$$C_2 = 4$$

$$\boxed{\therefore y = 4 \sin 6x}$$